

VI. Implementation of Quantum Computing

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1. Requirements for quantum computing architectures

The five DiVincenzo criteria:

- i.) A scalable physical system with well characterized qubits
 - ii) The ability to initialize the qubits to a simple fiducial state, such as $|000\dots\rangle$
 - iii) Long relevant decoherence times, much longer than the gate operation time
 - iv) A "universal" set of quantum gates
 - v) A qubit specific measurement capability
- i) A scalable physical system with well characterized qubits

▷ qubits: two level quantum system

• true two level system:

- spin $\frac{1}{2}$ particle (electron, nucleus, ...)

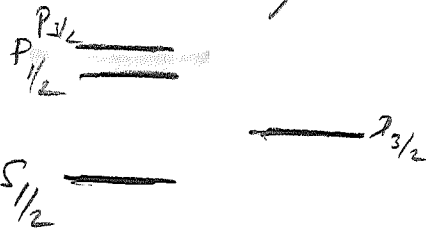
- photon polarization

→ other deg. of freedom (e.g. neutron) have trivial product structure!

• subspace of large system

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• 2 nat. states (energy levels) of atom/ion:



• 0 or 1 photons (or other bosons)

• current \uparrow or \downarrow

Advantage: extra levels \rightarrow extra control
(e.g. for initialization, readout, gates)

Disadvantage: Qubit loses to extra levels

\triangleright quantum bits: superpositions possible

e.g. $\left| \begin{array}{c} e^- \\ \cdot \\ \cdot \end{array} \right\rangle = |0\rangle$

$\left| \begin{array}{c} e^- \\ \cdot \end{array} \right\rangle = |1\rangle$

$\rightarrow \alpha|0\rangle + \beta|1\rangle$ ok \iff qubit

but: $\left| \begin{array}{c} \cdot \\ \cdot \end{array} \right\rangle = |0\rangle$

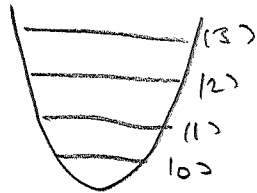
$\left| \begin{array}{c} e^- e^- \\ \cdot \cdot \end{array} \right\rangle = |1\rangle$

$|0\rangle + |1\rangle$ violates charge/particle # conservation $\rightarrow \text{no}$

▷ well characterized:

two level system protected from leakage

e.g. harm. oscillator $|0\rangle$ or $|1\rangle$



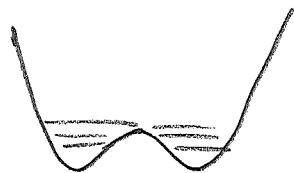
$$\Delta E = E_{L+1} - E_L = \hbar\omega$$

\Rightarrow equidistant

\Rightarrow transition $|0\rangle \leftrightarrow |1\rangle$ in resonance w/
 $|1\rangle \leftrightarrow |2\rangle$ etc.

\Rightarrow qubit not leak to higher levels when rotating!

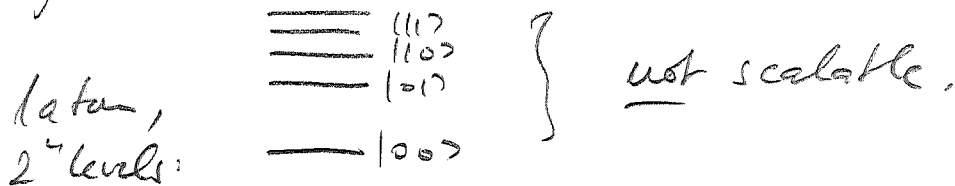
Better: non-quadr. potential:



\rightarrow levels not equidist.

▷ scalable: can put several qubits together.

E.g.



n atoms,
 2 levels:



Qubit could even live in some "encoded subspace", (163)

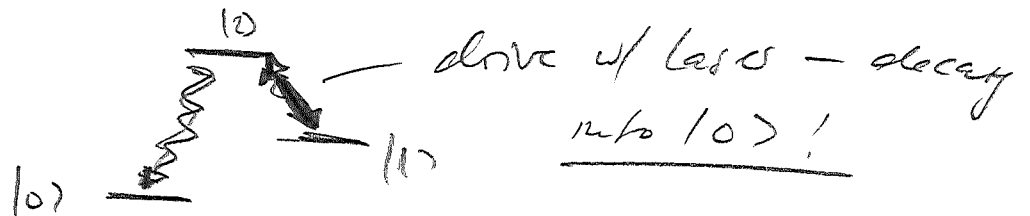
e.g. the $S = \frac{1}{2}$ space of 3 spin $\frac{1}{2}$ particles
(advantageous when symmetries exist/protect from noise)

(\rightarrow QECCs')

ii) The ability to initialize the qubits to a simple fiducial state, such as $|000\dots\rangle$

initialize to $|0\dots0\rangle$:

- equilibrate in ext field ΣB_z at low T.
- = optical pumping:



- non-destructive measurement

might involve creating qubit, e.g. when using photons:

\rightarrow need controlled single photon source!

Why initialization?

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→ Initial state at beginning of computation!

→ Error correction requires fresh ancillas

(or non-destructive ent. measurements...)

⇒ Noise introduces entropy. QECC reduces entropy ⇒ ancillas used to "dispose of entropy"

⇒ Need fast initialization

(faster than time scale of fastest noise!)

⇒ Equilibration slower than (class) noise!

Are mixed states ok?

→ Initialization into pure states (almost) impossible.

→ Mixed states ok?

e.g. $|\psi\rangle \rightarrow \rho = (1-p)|\psi\rangle\langle\psi| + p \mathbb{I}$

⇒ ok if p not too large:

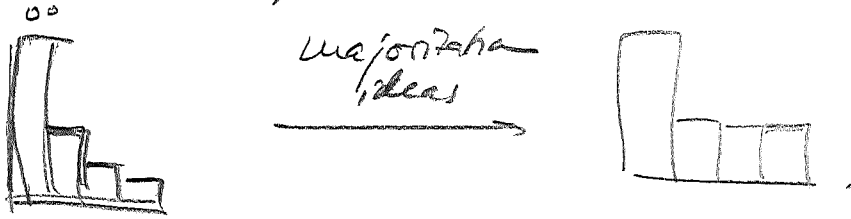
$$U \rho U^\dagger = (1-p) U|\psi\rangle\langle\psi|U^\dagger + p \mathbb{I}$$

\Rightarrow same outcome as $u/4$ (w/prob. $1-p$) 165

\Rightarrow same statistics.

(works as long as $1-p \geq \frac{1}{poly(n)}$)

Can transform any noise to \mathbb{I} :



Possible to "distill purity" (cf. ent. distillation):

$$p|0\rangle\langle 0| + (1-p)|1\rangle\langle 1|$$

2 qubits \Rightarrow

$p^2: 00\rangle$		$ 00\rangle: p^2$	
$p(1-p) \left\{ \begin{array}{l} 01\rangle \\ 10\rangle \end{array} \right.$	$\xrightarrow{\text{CNOT}}$	$\left. \begin{array}{l} 01\rangle \\ 10\rangle \end{array} \right\}$	qubit 2: max. mixed
$(1-p)^2: 11\rangle$		$ 11\rangle: (1-p)^2$	

\Rightarrow more pure:

$$\frac{p^2|0\rangle\langle 0| + (1-p)^2|1\rangle\langle 1|}{p^2 + (1-p)^2}$$

(iii) Long relevant decoherence times, much longer
than the gate operation time

- only local or relevant DoF relevant
(e.g. minimal DoF of trapped, or mostly irrelevant)
- error correction + threshold theorem: fixed retro
 $\frac{\text{decoh. time}}{\text{gate time}}$ sufficient $\sim 10^3$ to 10^4 gates (?) dep
on err. correction model - indep. of size of
computation.
- error vs. gate/ctrl./ meas. precision: can "bluff"
errors
- systematic errors can be systematically undone
(e.g. NMR: "always on" interactions - can
be cancelled)
- not too much correlations in random errors!

Two ess. types of errors:

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Dephasing: $|0\rangle + e^{i\phi}|1\rangle$
(Decoherence)

$\Rightarrow \phi$ randomized

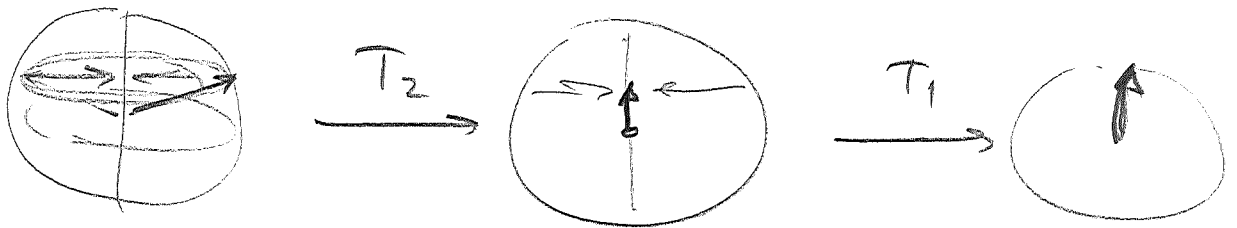
$\Rightarrow T_2$ time.

Depolarization / Relaxation:

$\alpha|0\rangle + \beta|1\rangle \longrightarrow |0\rangle$

$\Rightarrow T_1$ time

(only if $|0\rangle$ is preferred state, of course!)



By def.: $T_2 < 2T_1$, but typ. $T_2 \ll T_1$.

T_2 : relevant for gate operation.

T_1 : relevant for (z) measurement.

iv) A universal set of quantum gates

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→ Single qubit gates: $U = e^{-iHt/\hbar}$

→ Rotations in 2 bases

→ often continuous: $U_t = e^{-iHt/\hbar}$

⇒ precise control of t needed!

→ Two qubits gates:

→ one gate (not SWAP) sufficient!

→ best gate architecture - dep.:

$$H = \sigma_x \otimes \sigma_x, \sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y, \vec{\sigma} \cdot \vec{\sigma}$$

⇒ all conv. for right time t !

⇒ ideal gate tailored to architecture!

→ Symmetries / constraints:

→ gates can act in effective subspace,

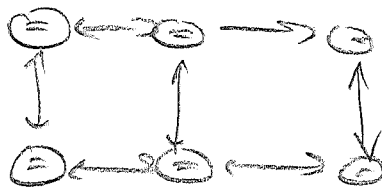
e.g. if H restricted by symmetries

($SU(2)$, transl. invariance)

→ interactions can be always or (decoupled)
by single-qubit ops.

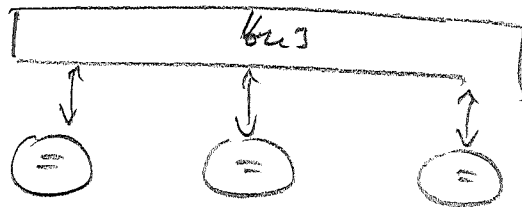
→ Interaction geometry:

Lattices:



- dep. on exp. setup ($\leq 3D$)
- e.g. solid state

bus:



bus = delocalized quantum PoT

which couples to all qubits

- need rel. control over couplings
- bus more sensitive for bigger systems

e.g.: ion traps: bus = joint vibrations of ions

move qubits:

e.g. trapped ions/atoms can be moved on "conveyor belts" (ew- fields or light fields)

v) A qubit specific measurement capability

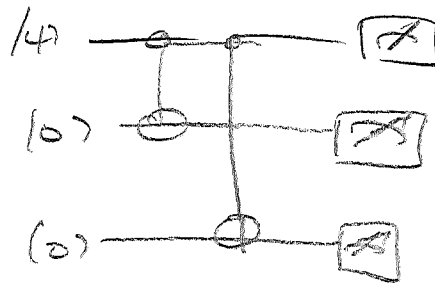
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Ideally: Projective meas. in some (2) basis.

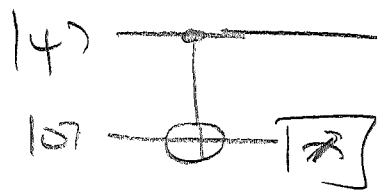
Imperfect measurement:

▷ repeat n

▷ copy + measure several times



Destructive meas. is ok: E.g. via copy:



\Rightarrow non-destructive!

Meas. on 1 qubit is ok: Can swap qubits.

Can be slow ($\sim T_1$) for final meas., but fast ($\ll T_2$) for error correction.

(Note: QECC does not require meas., but computing correction might be much faster on classical computers.)

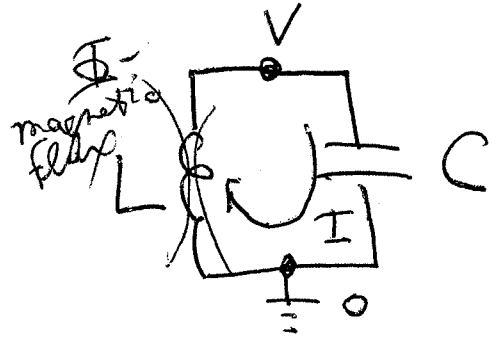
SUPERCONDUCTING QUBITS

- BASIS IN ELECTRIC CIRCUIT THEORY

1.

$$\Phi = -L I$$

$$\dot{\Phi} = V \quad (\text{Faraday})$$



$$(*) \quad Q = CV = C\dot{\Phi}$$

$$\dot{Q} = I = C\ddot{\Phi}$$

$$\Rightarrow \boxed{C\ddot{\Phi} = -\frac{1}{L}\Phi} \quad \text{equation of motion}$$

Lagrange function $\mathcal{L}(q, \dot{q})$ → resonates @ $\omega_0 = \frac{1}{\sqrt{LC}}$
 (conservative mechanics)

equation of motion from Lagrangian:

Euler-Lagrange equation:

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}} = \frac{\partial \mathcal{L}}{\partial q}$$

good guess

$$\mathcal{L} = \frac{1}{2} C (\dot{\Phi})^2 - \frac{1}{2L} \Phi^2$$

electric energy

magnetic energy

$$q \Leftrightarrow \Phi$$

↑
canonical coordinate

prescription for obtaining Hamiltonian function:

$$p \equiv \frac{\partial \mathcal{L}}{\partial \dot{q}} = \frac{\partial \mathcal{L}}{\partial \dot{\Phi}} = C \dot{\Phi}$$

C like a mass

Legendre transformation

$$H = \dot{q}p - \mathcal{L}, \text{ plus replace } \dot{q} = f(p)$$

$$\Phi = \frac{1}{C} p$$

NB $p = Q$
from (*)

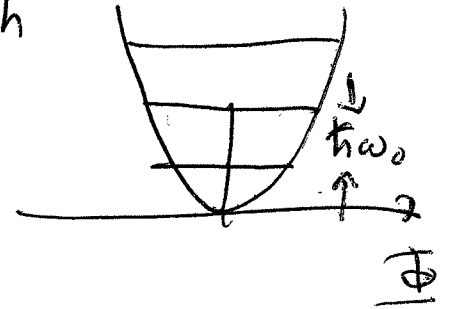
$$H = \frac{1}{2} C \dot{\Phi}^2 + \frac{1}{2L} \Phi^2$$

2.

$$H = \frac{Q^2}{2C} + \frac{1}{2L} \Phi^2$$

quantum postulate: $[\hat{Q}, \hat{\Phi}] = -i\hbar$

$$\hat{H} = \frac{-\hbar^2}{2C} \frac{\partial^2}{\partial \Phi^2} + \frac{1}{2L} \Phi^2 \rightarrow$$



To make a qubit, make non-harmonic

⇒ nonlinear inductance

$$I = \frac{1}{L} \Phi \quad \text{linear}$$

= $f(\Phi)$ nonlinear → still non-dissipative

particular nonlinearity in superconducting tunneling device:

Josephson Junction:



$$I = \frac{\tilde{\Phi}_0}{L_J} \sin(\Phi / \tilde{\Phi}_0)$$

$$\tilde{\Phi}_0 = \frac{\hbar}{2e}$$

← "Josephson inductance"

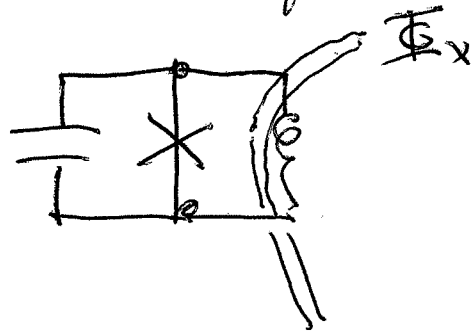
$$\frac{\tilde{\Phi}_0}{L_J} = I_c$$

critical current

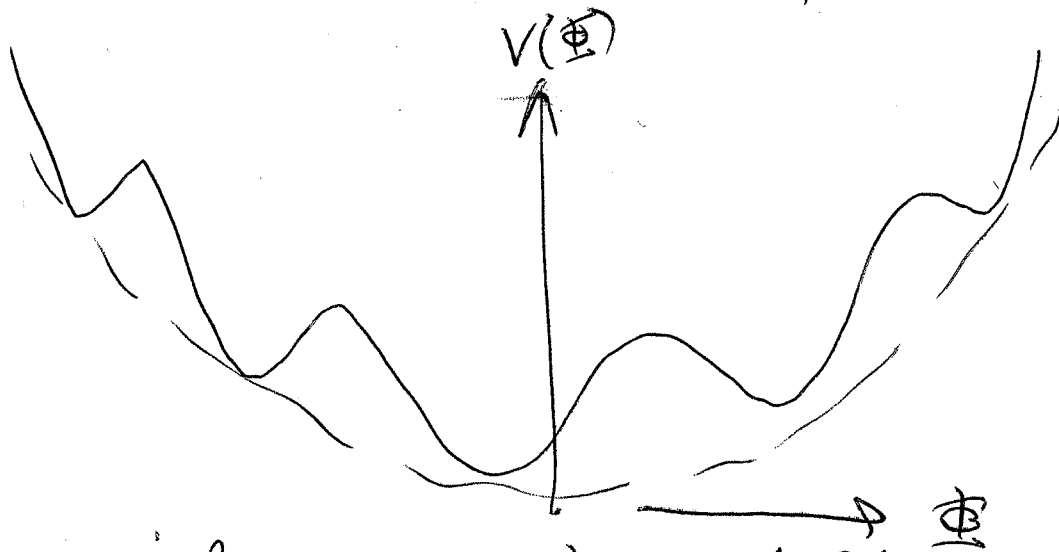
← usually just \hbar .

So, eq. Hamiltonian for

3.



$$\hat{H} = \frac{-\hbar}{2C} \frac{\partial^2}{\partial \Phi^2} + \frac{\tilde{\Phi}_0^2}{L_J} \cos\left(\frac{\Phi}{\tilde{\Phi}_0}\right) + \frac{L}{2L} \Phi^2 + \frac{\Phi \Phi_x}{L} \quad (*)$$



Highly non-harmonic potential!

Linear term (*) \rightarrow characteristic "phase rigidity" — characteristic of superconductivity (not true of "perfect conductor")

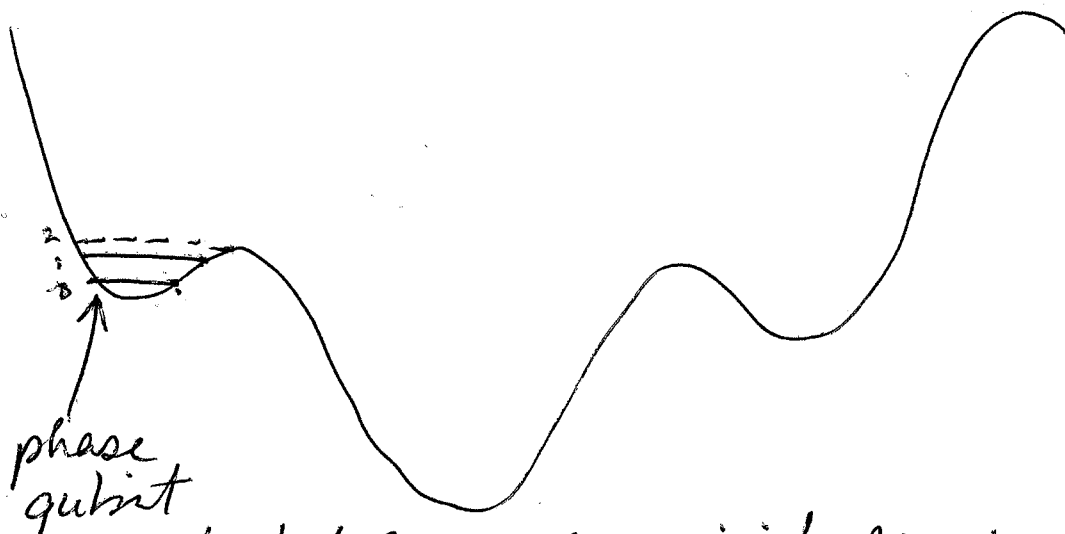


MORE GENERAL THEORY FOR DERIVING CIRCUIT HAMILTONIAN:

Network Graph Theory.

Applications:

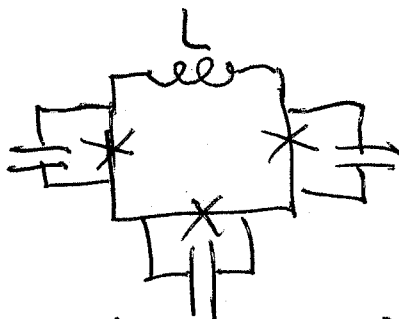
1. Phase qubit - Eq. (*) on p. 3.



- use metastable levels - initialize by ramping Φ_x
- read out by pumping to metastable state $|2\rangle$, observe "escape"

2. Flux qubit

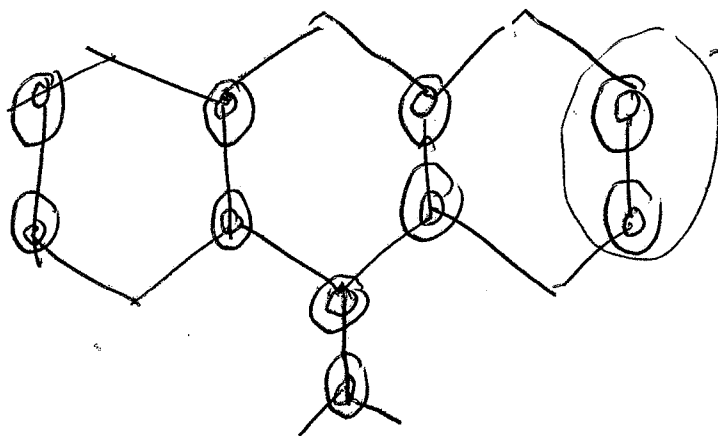
3D Hamiltonian



$$U(\Phi) \approx \frac{\Phi_0^2}{2} \left[\sum_{i=1}^3 L_{J,i} \cos \varphi_i + \frac{1}{2L} (\varphi_1 + \varphi_2 + \varphi_3)^2 + \frac{1}{L} (\varphi_1 + \varphi_2 + \varphi_3) \frac{\Phi_x}{\Phi_0} \right]$$

$\langle 111 \rangle$ direction very tightly confined.

- Potential in plane \perp to $\langle 111 \rangle$:

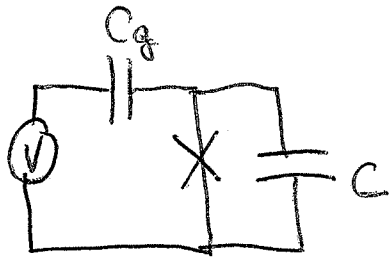


very controllable double-well potential

TRANSMON QUBIT

& COUPLED SUPERCONDUCTING QUBITS.

1.



see:

www.qipc2011.ethz.ch/uploads/schoolpresentations/qipcAlexandre.pdf

$$H = \frac{1}{2(C+C_g)} (\hat{Q} - C_g V)^2 - \left(\frac{\Phi_0}{2\pi}\right)^2 \frac{1}{L_J} \cos \hat{\varphi}$$

$$= \underbrace{\frac{(2e)^2}{2(C+C_g)}}_{4E_c} \left(\hat{n} - \underbrace{\frac{C_g V}{2e}}_{n_{\text{offset}}} \right)^2 - \underbrace{\left(\frac{\Phi_0}{2\pi}\right)^2 \frac{1}{L_J}}_{E_J} \cos \hat{\varphi} \quad \hat{n} = -i \frac{\partial}{\partial \varphi}$$

$$H = 4E_c (\hat{n} - n_{\text{offset}})^2 - E_J \cos \varphi$$

NOTE φ is only defined on interval $0 \leq \varphi < 2\pi$.

Periodic boundary conditions if $n_{\text{offset}} = 0$
(or integer)

$n_{\text{offset}} \neq 0$ acts like Aharonov-Bohm flux

For $E_J = 0$, ~~the~~ eigenfunctions are

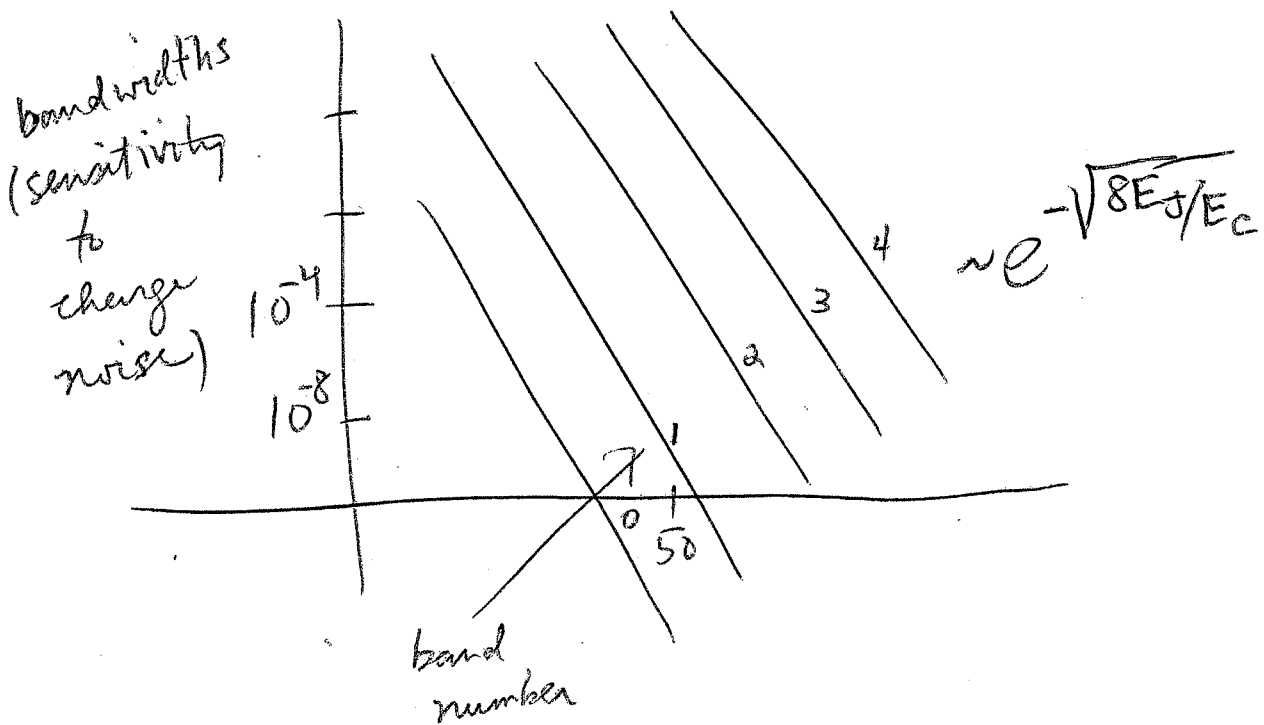
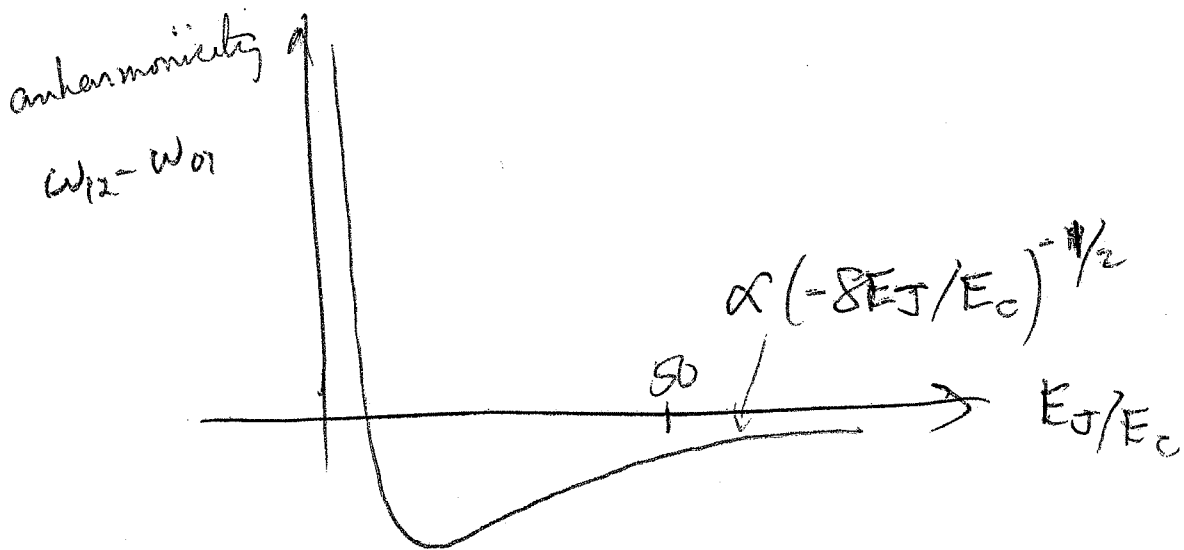
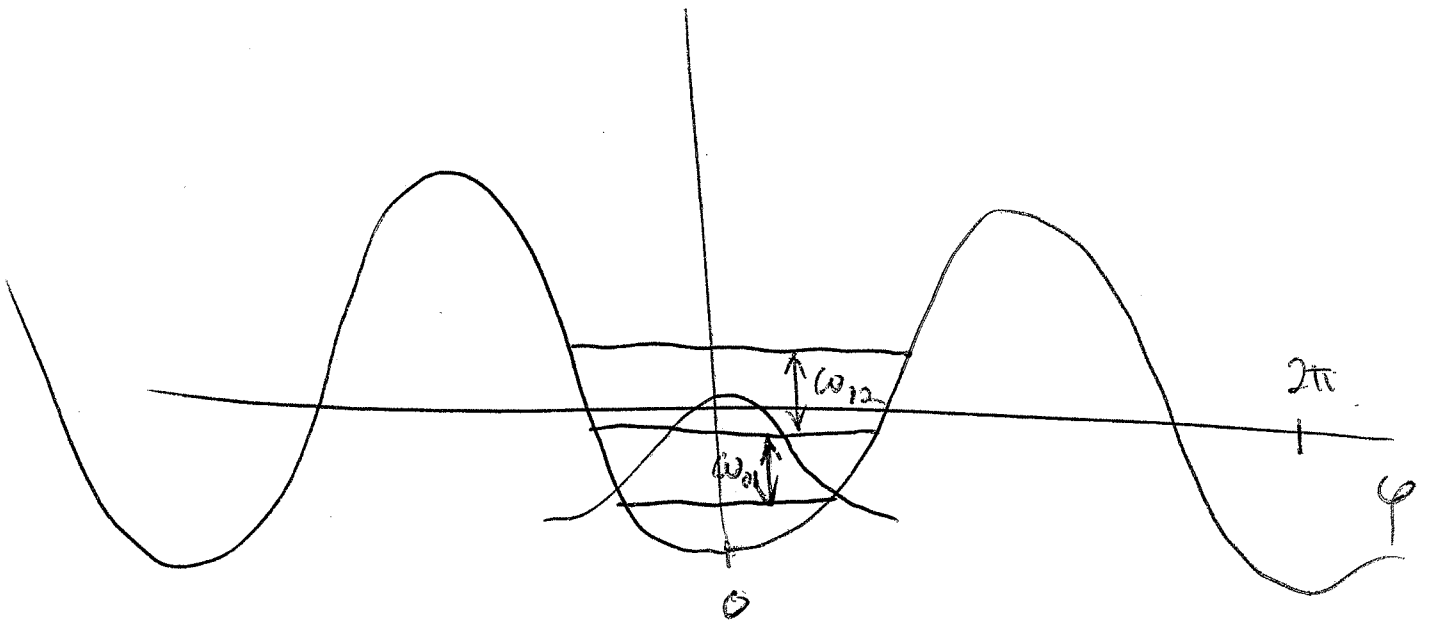
plane waves:

$$H|\psi\rangle = E|\psi\rangle$$

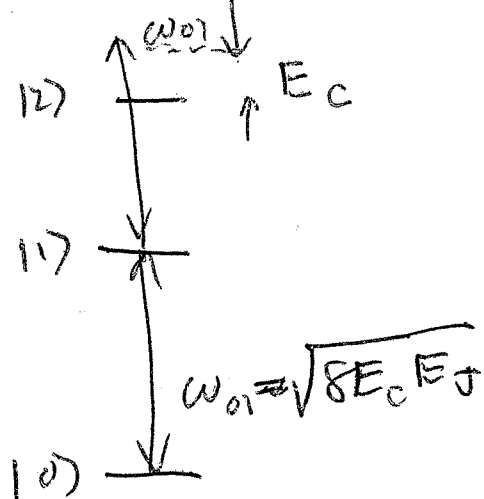
$$|\psi\rangle = e^{im\varphi}$$

$$m = 0, \pm 1, \pm 2, \dots$$

$$E = 4E_c (m - n_{\text{offset}})^2$$



i.e., charge sensitivity exponentially suppressed, anharmonicity "reasonable". 4.

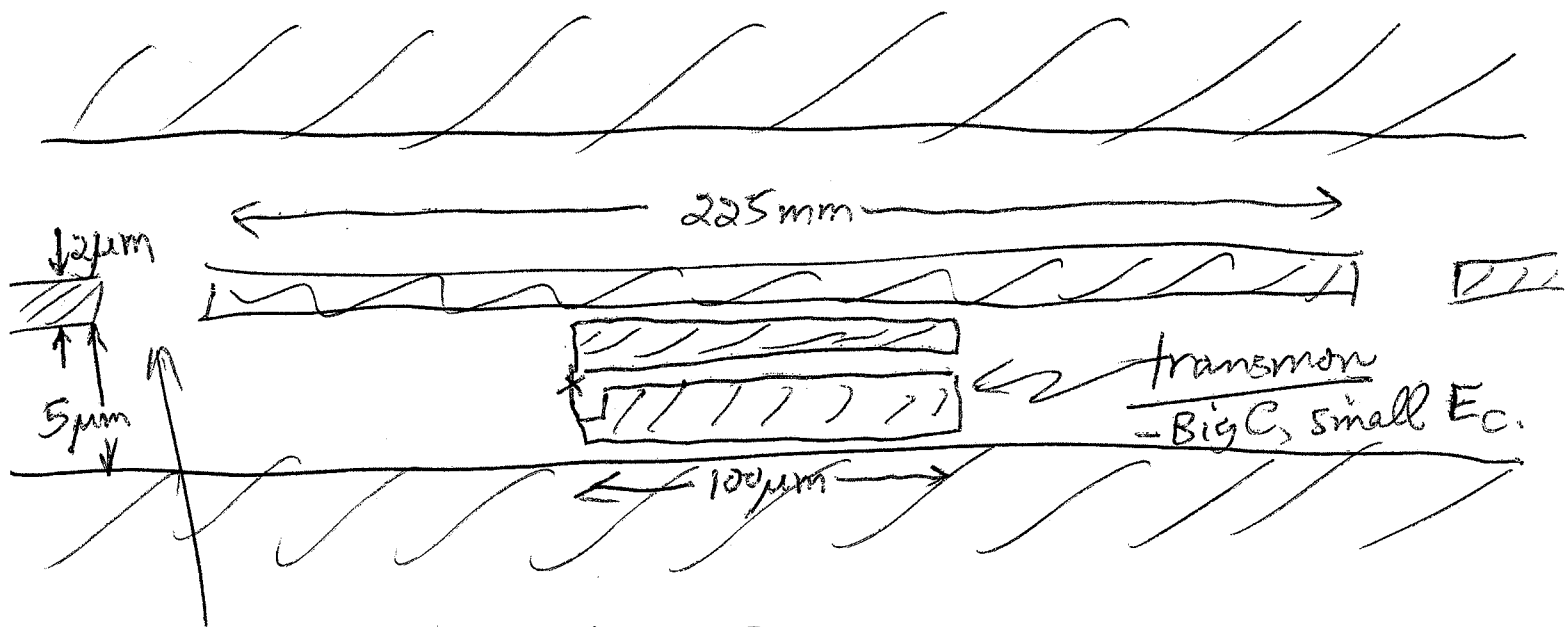


- Very accurate 1-qubit gates,

$$F = 99.993\%$$

- COUPLING TRANSMONS.

coplanar waveguide:



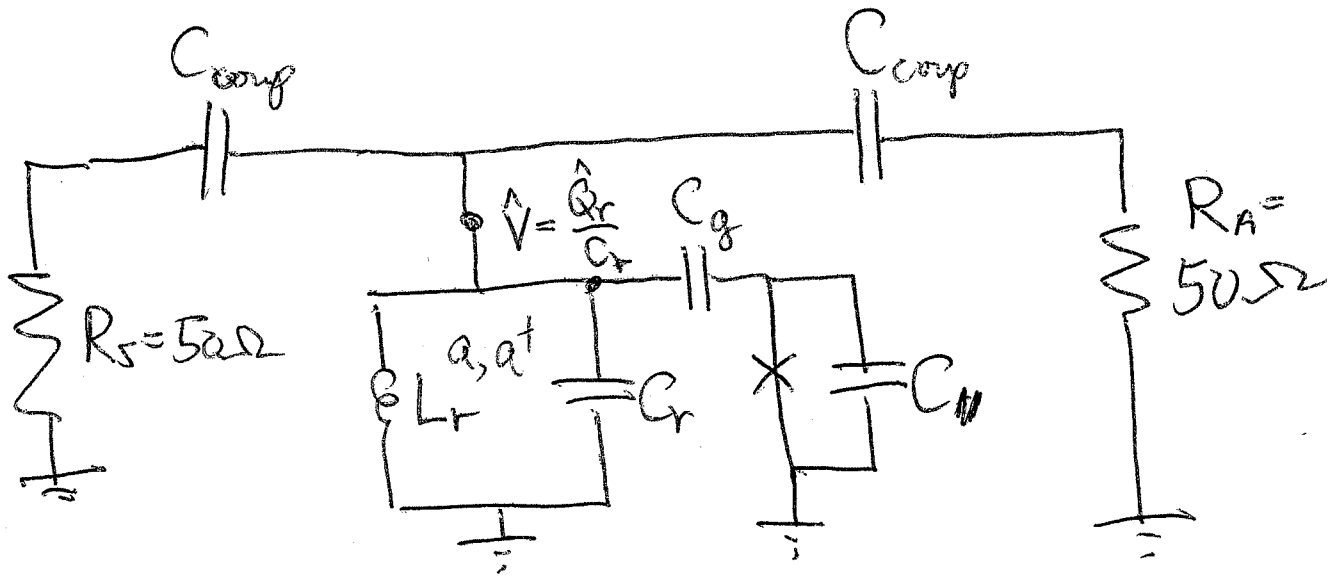
coupling capacitors C_{coup} .

→ slightly transparent mirror for microwaves

CPW is cavity for microwave photons, $\frac{\omega_c}{2\pi} = 2-20$ GHz.

Circuit model:

5.



R_S, R_A : source & amplifier impedances

L_r, C_r : model for first resonant mode of transmission line resonator

circuit Hamiltonian

$$H = \frac{\hat{Q}_r^2}{2C_r} + \frac{\Phi_r^2}{2L_r} + \left[4E_c (\hat{n} - \hat{n}_{offset})^2 - E_J \cos \hat{\varphi} \right]$$

$$\hat{n}_{offset} = \frac{C_g}{2e} \hat{V} = \frac{C_g}{2eC_r} \hat{Q}_r \quad \leftarrow \text{offset-charge operator}$$

2nd quantize: coupling term $-8E_c \hat{n} \hat{n}_{offset}$

rotating-wave approx.

$$H = \omega_r a^\dagger a + \frac{\omega_{01}}{2} Z + g (a \sigma_- + a^\dagger \sigma_+)$$

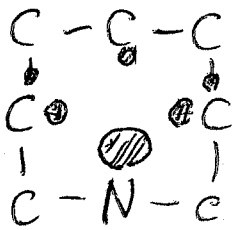
Jaynes-Cummings model:

SPIN QUBITS

- There is a wide variety of spin-qubits.
- We mean electron spin qubits.
- We discuss several cases, and consider "which electron". Because electrons are indistinguishable, we really mean "which fermionic mode"

EXAMPLES

1 NV⁻ center (diamond)



HOMO
highest occupied molecular orbital

2 of the 3 states of a triplet

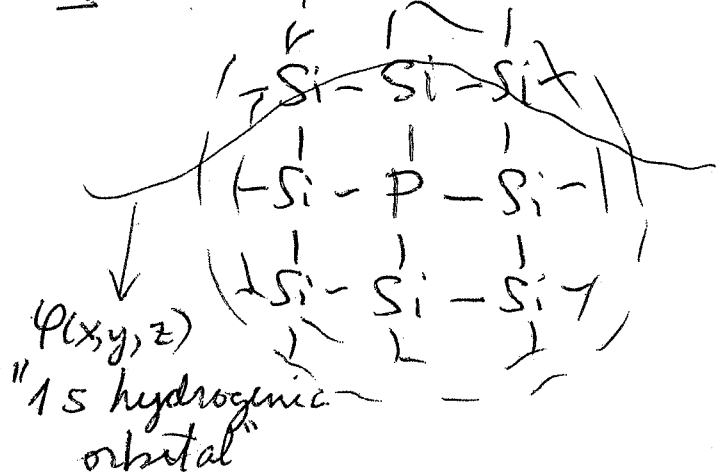
$$\left\{ \begin{array}{l}
 C_{\text{HOMO}(1)}^{\uparrow} C_{\text{HOMO}(2)}^{\uparrow} \\
 C_{\text{HOMO}(1)}^{\downarrow} C_{\text{HOMO}(2)}^{\downarrow}
 \end{array} \right\} \uparrow \downarrow \prod_k C_k^{\uparrow} |0\rangle$$

vacuum ↓

↑ "filling the bands"

- orbitals not varied by \vec{E} field

2 "Kane qubit" (B. Kane, 1998)



$$|0\rangle = C_{S\uparrow}^{\dagger} \prod_k C_k^{\dagger} |0\rangle$$

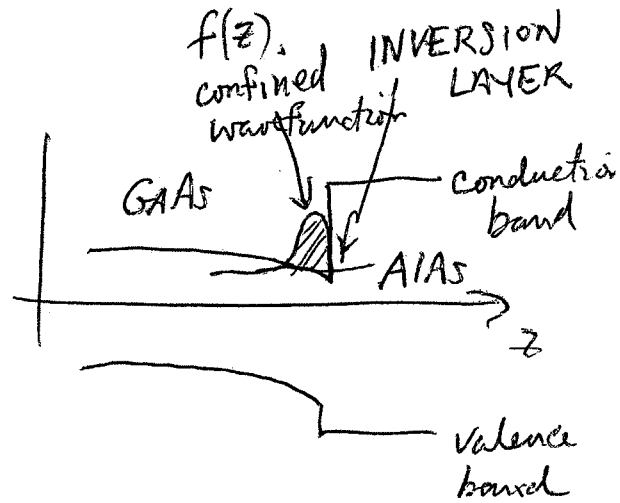
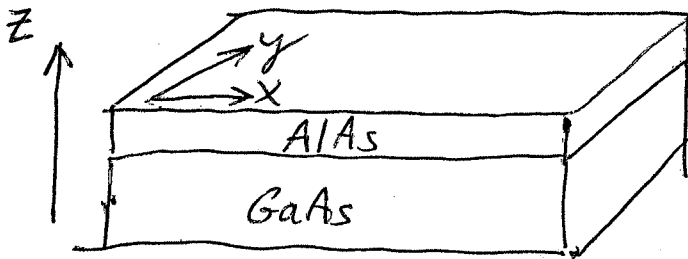
$$|1\rangle = C_{S\downarrow}^{\dagger} \prod_k C_k^{\dagger} |0\rangle$$

- orbital can be distorted by \vec{E}

field (large Stark effect)

GaAs QUANTUM-DOT QUBIT

2.



- SIMPLE, CUBIC CRYSTAL STRUCTURE
- defect-free interface
- electrons strongly confined in z-direction, completely free in x and y.

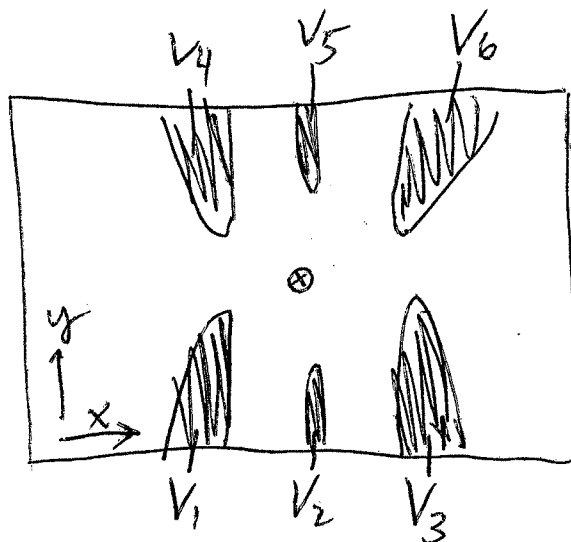
- full wavefunction:

$$\Psi(x, y, z) = f(z) e^{i\vec{k} \cdot (x, y)} u(x, y)$$

periodic function
↓ (crystal period)

2D Bloch theorem

MAKE A QUANTUM DOT BY FURTHER CONFINEMENT:



⊗ minimum of electric potential in interfacial plane

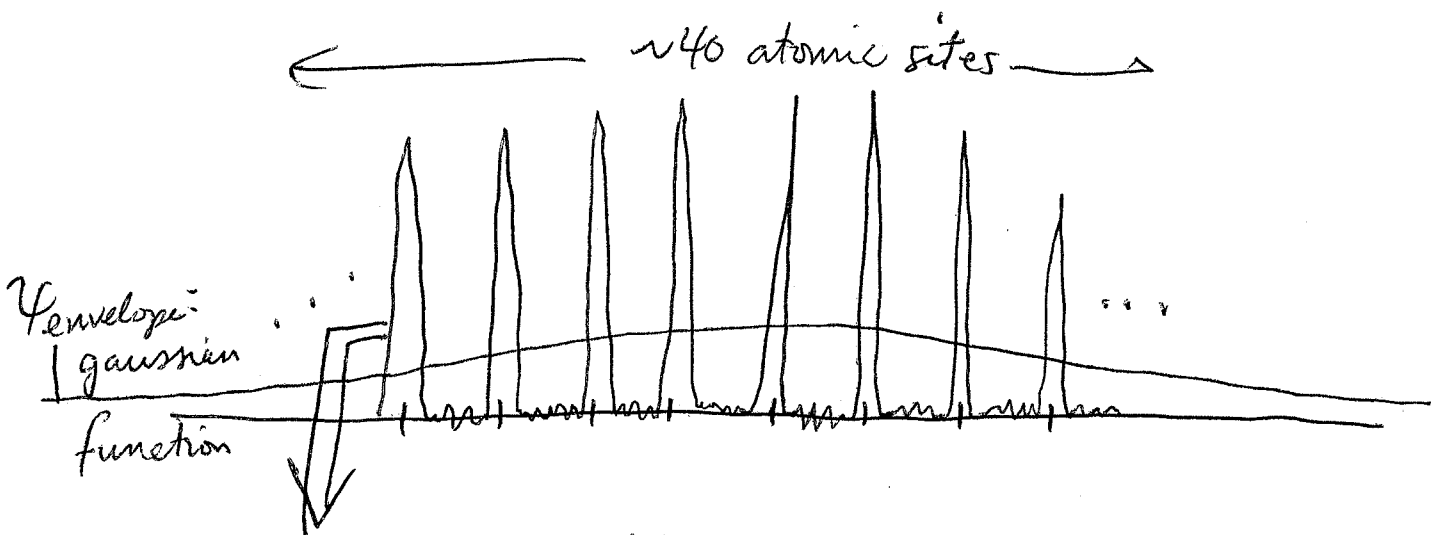
now the wavefunction is

3.

$$\Psi(x, y, z) = f(z) \Psi_{\text{envelope}}(x, y) u(x, y)$$

for GaAs, the lowest-energy conduction-band state is at $k=0$ ("direct bandgap").

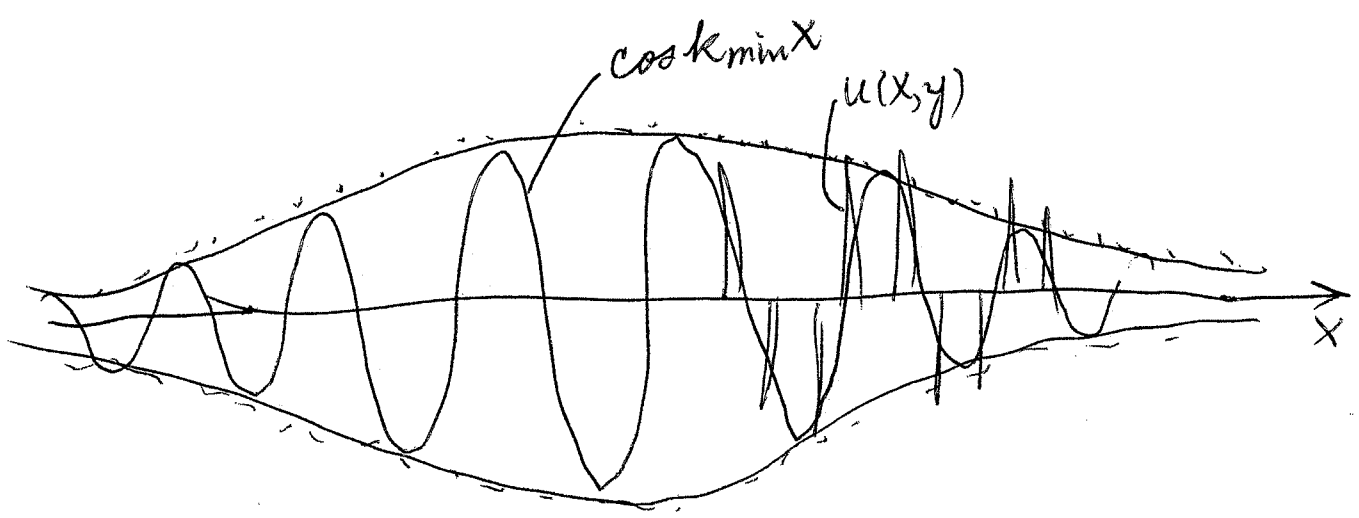
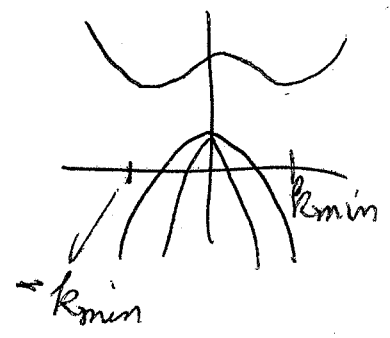
This leads to a Ψ_{envelope} which is very slowly varying, something like



Ψ_{envelope} multiplies the very sharply peaked $u(x, y)$. This function is very large at the site of the nuclei, giving these "concentration factors":

$$\left| \frac{\Psi(0)}{\Psi_{\text{envelope}}(0)} \right|^2 = \begin{cases} 2.7 \times 10^3 & \text{Ga} \\ 4.5 \times 10^3 & \text{As} \\ 186 & \text{Si} \end{cases}$$

in silicon heterostructures, the wavefunction is less spread out, because the effective mass is heavier. The other complication of Si is that the conduction band minimum is not at $k=0$ ("indirect bandgap"). This causes the envelope function to be more complex:



$g(x) \cos k_{min} x$ and $g(x) \sin k_{min} x$ are two different, almost degenerate wavefunctions. This is "valley degeneracy" (also in y direction), which is not a desirable extra quantum number for a qubit.

NUCLEAR SPINS

^{69}Ga	60%	$I = 3/2$
^{71}Ga	40%	$I = 3/2$
^{75}As	100%	$I = 3/2$
^{28}Si	~90%	$I = 0$
^{30}Si	~6%	$I = 0$
^{29}Si	~3%	$I = 1/2$
^{12}C	99%	$I = 0$
^{13}C	1%	$I = 1/2$
^{229}Th	?	$I = 5/2$ $I = 3/2 \leftarrow$ extra

^{14}N	99.6%	$I = 1$
^{15}N	0.4%	$I = 1/2$

Amplification factor is important because of Fermi contact

hyperfine interaction:

$$H_{\text{SB}} = \sum_{i=1}^{10000} A |\psi(\vec{r}_i)|^2 \vec{S} \cdot \vec{I}_i$$

↑ electron spin ↑ nuclear spin

if all nuclear spins are polarized, $I_i = -\frac{3}{2}$,

the quantity

$$\sum_{i=1}^{10000} A |\psi(\vec{r}_i)|^2 \left(-\frac{3}{2}\right)$$

is like a z magnetic field of 5T.

(Overhauser field)

7.8 ± 0.5 eV above ground state

N.B. of the three terms in the

6.

hyperfine hamiltonian, the "longitudinal term"

ZI_z is 1st order in perturbation theory,

the "transverse terms" $XI_x + YI_y$ are

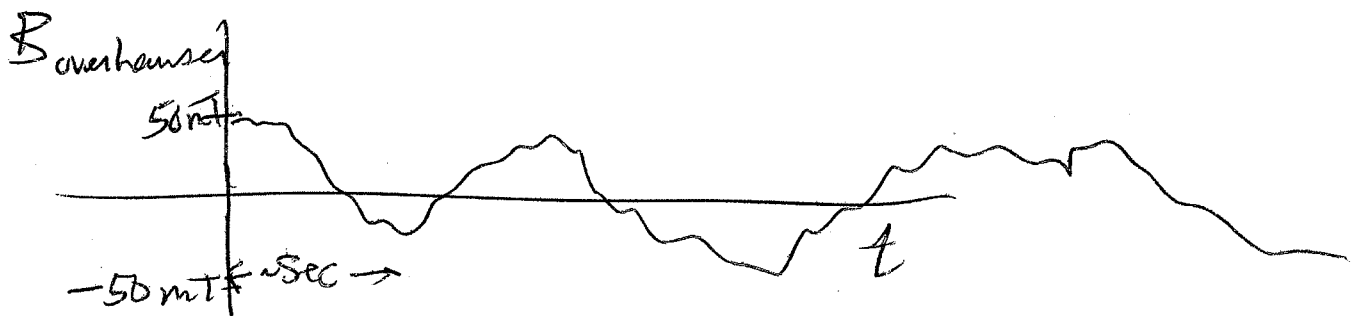
2nd order. The transverse terms also

disappear in a rotating wave approximation.

Actual situation with nuclear spins:

Z-magnetization is a fluctuation around a

zero mean, with amplitude $O(\sqrt{N})$: $\frac{5T}{\sqrt{N}} \sim 50\text{mT}$



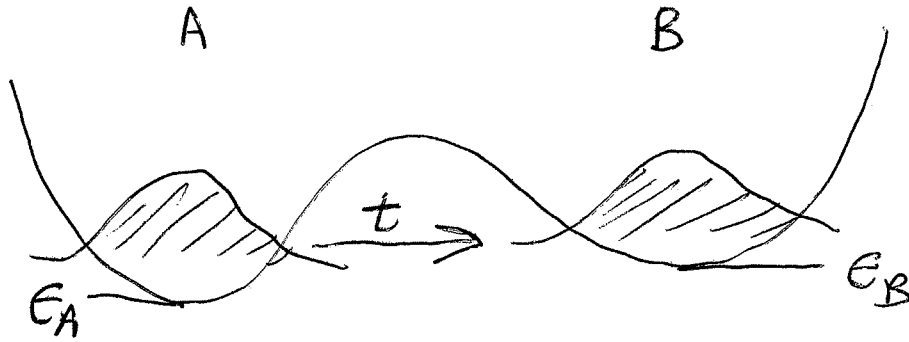
$T^* \sim 15\text{mT}$, can be effectively refocused.

NEXT: 2 quantum dots.

SPIN QUBITS - CONTINUED.

1.

- TWO COUPLED SPINS:



2nd QUANTIZED HAMILTONIAN:

$$H = E_A (a_{\uparrow}^{\dagger} a_{\uparrow} + a_{\downarrow}^{\dagger} a_{\downarrow}) + E_B (b_{\uparrow}^{\dagger} b_{\uparrow} + b_{\downarrow}^{\dagger} b_{\downarrow}) + t (a_{\uparrow}^{\dagger} b_{\uparrow} + b_{\uparrow}^{\dagger} a_{\uparrow} + a_{\downarrow}^{\dagger} b_{\downarrow} + b_{\downarrow}^{\dagger} a_{\downarrow}) + U_A a_{\uparrow}^{\dagger} a_{\uparrow} a_{\downarrow}^{\dagger} a_{\downarrow} + U_B b_{\uparrow}^{\dagger} b_{\uparrow} b_{\downarrow}^{\dagger} b_{\downarrow}$$

Work in Fock space of 2 electrons in these 4 modes

$| \uparrow_{a_{\downarrow}} \uparrow_{a_{\uparrow}} \uparrow_{b_{\downarrow}} \uparrow_{b_{\uparrow}} \rangle \quad \binom{4}{2} = 6$ Dimensional space

$$H = \begin{array}{cccccc} & 0101 & 0110 & 1001 & 1010 & 1100 & 0011 \\ 0101 & E_A + E_B & & & & 0 & 0 \\ 0110 & & E_A + E_B & & & -t & -t \\ 1001 & & & E_A + E_B & & t & t \\ 1010 & & & & E_A + E_B & 0 & 0 \\ \hline 1100 & 0 & -t & t & 0 & U_A + 2E_A & 0 \\ 0011 & 0 & -t & t & 0 & 0 & U_B + 2E_B \end{array}$$

A non-trivial 2-qubit gate requires that 2.
 we include the effects of the non-qubit
 states $|1100\rangle$ and $|1001\rangle$

- Sign of t :

These signs arise from the antisymmetry
 of the Fermionic wavefunction. Such
 signs are ~~not~~ inevitable, since if we call

$$|0110\rangle \equiv a_{\uparrow}^{\dagger} b_{\downarrow}^{\dagger} |0000\rangle$$

then $b_{\downarrow}^{\dagger} a_{\uparrow}^{\dagger} |0000\rangle = -|0110\rangle$, because of
 the anticommutation relations of the
 creation and destruction operators. ~~These~~

Everything works properly if you use this
 standard, general convention for the sign:

$$c_x^{\dagger} |b_1, b_2, \dots, \underbrace{b_{x-1}}_{\substack{\uparrow \\ \text{mode } x}}, \dots \rangle = (-1)^{(b_1 + b_2 + \dots + b_{x-1}) \bmod 2} |b_1, b_2, \dots, b_{x-1}, \boxed{1}, \dots \rangle$$

$$c_x |b_1, b_2, \dots, \boxed{1}, \dots \rangle = (-1)^{(b_1 + b_2 + \dots + b_{x-1}) \bmod 2} |b_1, b_2, \dots, b_{x-1}, \boxed{0}, \dots \rangle$$

With this the evaluation of the matrix elements gives 2 types of cases

$$\langle 1, \overset{\text{no electron "in the way"}}{\downarrow} 0, 0, \phi | H | 0, \overset{\downarrow}{0}, 1, \phi \rangle = +t$$

$$\langle 1, \overset{\downarrow}{1}, 0, 0 | H | 0, \overset{\downarrow}{1}, 1, 0 \rangle = -t.$$

Interpreting our 6x6 Hamiltonian.

- Basis change

notation

$$|S\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

$$\begin{aligned} \uparrow\downarrow &\Leftarrow 0110 \\ \downarrow\uparrow &\Leftarrow 1001 \end{aligned}$$

$$|T_0\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$

$\uparrow\uparrow$	0	0	NB: I have shifted the zero of energy by $E_A + E_B$.			
S	$-\sqrt{2}t$	$-\sqrt{2}t$				
T_0	0	0				
$\downarrow\downarrow$	0	0				
1100	0	$+\sqrt{2}t$	0	0	$U_A - E_B + E_A$	0
0011	0	$-\sqrt{2}t$	0	0	0	$U_B + E_B - E_A$

2nd order perturbation theory shifts the eigenvalue of $|S\rangle$ down by

$$\langle S | H | S \rangle = \frac{-2t^2}{U_A + E_A - E_B} - \frac{2t^2}{U_B + E_B - E_A} \equiv J$$

truncated Hamiltonian:

4.

$$H = \begin{matrix} \uparrow\uparrow \\ \text{"S"} \\ T_0 \\ \downarrow\downarrow \end{matrix} \begin{pmatrix} 0 & & & \\ & J & 0 & \\ & & 0 & \\ 0 & & & 0 \end{pmatrix}$$

"|S>" contains some perturbative admixture of
|1100> and |0011>

- going to original basis:

$$H = \begin{matrix} \uparrow\uparrow \\ \text{"}\uparrow\downarrow\text{"} \\ \text{"}\downarrow\uparrow\text{"} \\ \downarrow\downarrow \end{matrix} \begin{pmatrix} 0 & & & \\ & \frac{1}{2}J & -\frac{1}{2}J & \\ & -\frac{1}{2}J & \frac{1}{2}J & \\ & & & 0 \end{pmatrix}$$

NB $XX + YY + ZZ - III =$

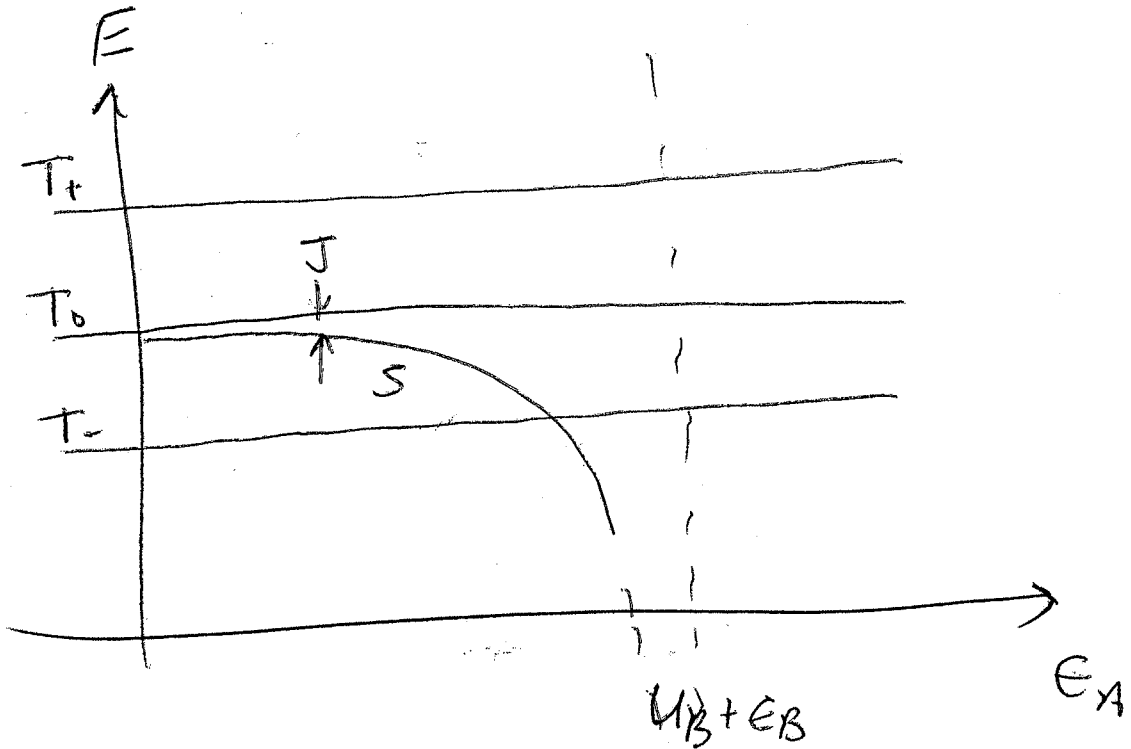
$$\begin{pmatrix} 0 & & & \\ & -2 & 2 & \\ & 2 & -2 & \\ & & & 0 \end{pmatrix}$$

thus, up to another shift,

$$H \propto J \vec{S}_A \cdot \vec{S}_B \quad \text{Heisenberg interaction}$$

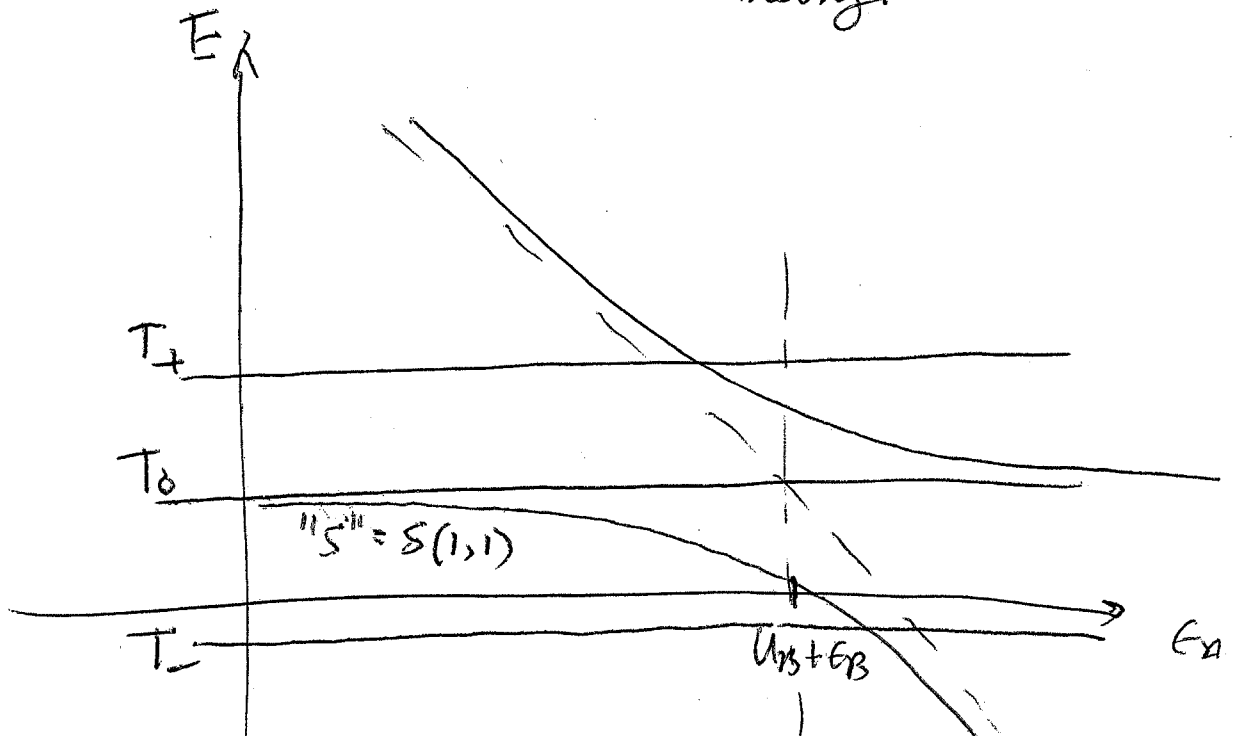
- consider J as a function of ϵ_A :

ADDING ZEEMAN FIELD TO BREAK ($T_{0\pm}$) 5.
DEGENERACY:



↑
divergence at this point -
artifact of perturbation
theory.

- DIAGONALIZE 6X6 MATRIX:

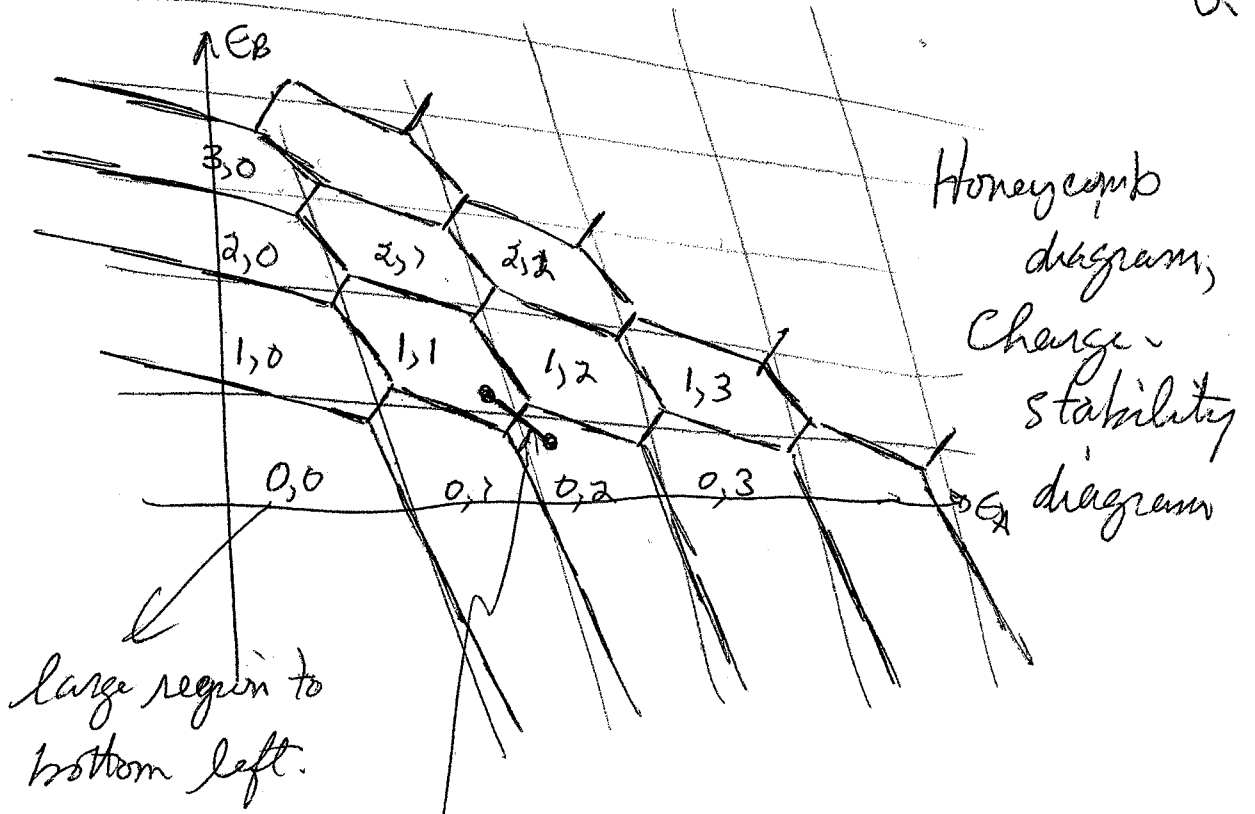


Smooth crossover to non-computational
singlet.

$S(0,2)$
 $= |0011\rangle$

MORE GLOBALLY:

6.



QUANTUM EXPERIMENTS

CHARGE SENSING — POINT CONTACT

