VI. Implementation of Quantum Computing

1. Requirements for quantum computing architectures

The five DiVincenzo criteria:

i) A scalable physical system with well characterized qubits

ii) The ability to initialize the qubits to a simple fiducial state, such as $|000\ldots\rangle$

iii) Long relevant decoherence times, much longer than the gate operation time

iv) A "universal" set of quantum gates

v) A qubit specific measurement capability

i) A scalable physical system with well characterized qubits

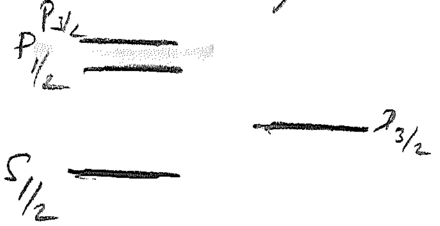
▷ qubits: two level quantum system

- true two level system:
  - spin $\frac{1}{2}$ particle (electron, nucleus, ...)
  - photon polarization

→ other deg. of freedom (e.g. motion) have tensor product structure!

- *subspace* of large system

  - 2 int. states (energy levels) of atom/ion:



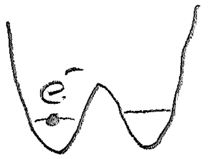
  - 0 or 1 photons (or other bosons)
  - current ↻ or ↺

Advantage: extra levels → extra control (e.g. for initialization, read-out, gates)

Disadvantage: Qubit loss to extra levels

▷ *quantum bits*: *superpositions* possible

e.g.

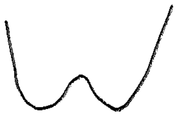


$= |0\rangle$

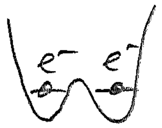
$= |1\rangle$

→ $\alpha|0\rangle + \beta|1\rangle$ ok → qubit

but:



$= |0\rangle$



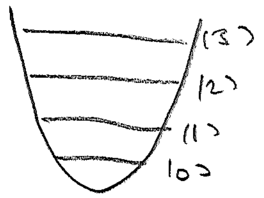
$= |1\rangle$

$|0\rangle + |1\rangle$ violates charge/particle # conservation → ↯

▷ <u>well characterized</u>:

two level system protected from leakage

e.g. harm. oscillator in $|0\rangle$ or $|1\rangle$



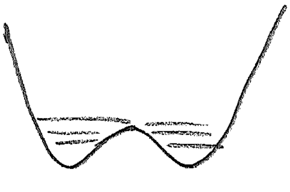
$\Delta E = E_{n+1} - E_n = \hbar\omega$

$\Rightarrow$ equidistant

$\Rightarrow$ transition $|0\rangle \longleftrightarrow |1\rangle$ in resonance w/ $|1\rangle \longleftrightarrow |2\rangle$ etc.

$\Rightarrow$ qubit will leak to higher levels when rotating!

Better: non-quadr. potential:

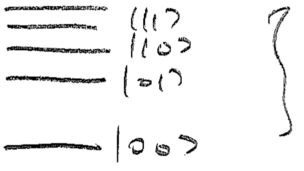


$\longrightarrow$ levels not equidist.

▷ <u>Scalable</u>: can put several qubits together.

E.g.

1 atom, 2^n levels:



<u>not</u> scalable.

n atoms, 2 levels:

: scalable.

Qubit could even live in some "encoded subspace",

e.g. the $S=\frac{1}{2}$ space of 3 spin $\frac{1}{2}$ particles

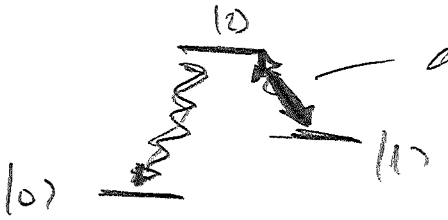
(advantageous when symmetries exist/protect from noise)

(→ QECCs!)

ii) The ability to initialize the qubits to a simple fiducial state, such as $|00.0\ldots\rangle$

Initialize to $|0\ldots0\rangle$:

- equilibrate in ext field $\sum \sigma_z$ at low T.
- optical pumping:



drive w/ laser – decay into $|0\rangle$!

- non-destructive measurement

Might involve creating qubit, e.g. when using photons:

→ need controlled single photon source!

Why initialization?

→ Initial state at beginning of computation!

→ Error correction <u>requires fresh ancillas</u>

(or non-destructive <u>ent.</u> measurements...)

⇒ Noise introduces entropy. QECC reduces entropy ⇒ ancillas used to "dispose of entropy"

⇒ Need <u>fast</u> initialization

(faster than time scale of fastest noise!)

→ Equilibration <u>slower</u> than (phase) noise!

Are mixed states ok?

→ Initialization into pure states (almost) impossible.

→ Mixed states ok?

e.g. $|\psi\rangle \rightarrow \rho = (1-p)|\psi\rangle\langle\psi| + p\,\mathbb{1}$

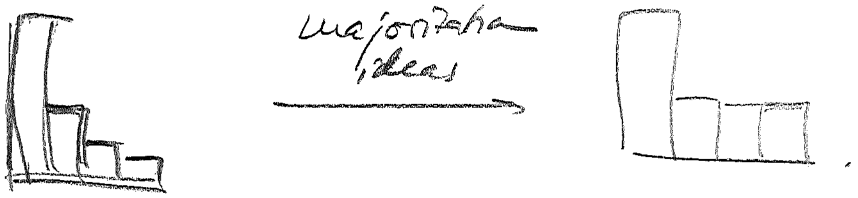
⇒ ok if p not too large:

$$U\rho U^+ = (1-p)\,U|\psi\rangle\langle\psi|U^+ + p\,\mathbb{1}$$

$\Rightarrow$ same outcome as $\mathcal{U}|\psi\rangle$ (w/ prob. $1-p$)

$\Rightarrow$ same statistics.
(Works as long as $1-p \geq \frac{1}{\mathrm{poly}(n)}$)

Can transform any noise to $\mathbb{1}$:



Possible to "distill purity" (cf. ent. distillation):

$$p|0\rangle\langle 0| + (1-p)|1\rangle\langle 1|$$

$$\text{2 qubits} \Rightarrow \begin{array}{ll} p^2: & |00\rangle \\ p(1-p) & \begin{cases} |01\rangle \\ |10\rangle \end{cases} \\ (1-p)^2: & |11\rangle \end{array} \xrightarrow{\text{CNOT}} \begin{array}{l} |00\rangle : p^2 \\ \left.\begin{array}{l} |01\rangle \\ |11\rangle \end{array}\right\} \text{qubit 2: max. mixed} \\ |10\rangle \quad (1-p)^2 \end{array}$$

$$\Rightarrow \text{more pure: } \frac{p^2|0\rangle\langle 0| + (1-p)^2|1\rangle\langle 1|}{p^2+(1-p)^2}$$

iii) Long relevante decoherence times, much longer than the gate operation time

→ only decoh. on relevant DoF relevant

(e.g. motional DoF of ~~trapped~~ ion mostly irrelevant)

→ error correction + threshold thm.: fixed ratio $\frac{\text{decoh. time}}{\text{gate time}}$ sufficient $\sim 10^3$-10^4 gates (?) dep. on err. correction model – indep. of size of computation.

→ error vs. gate/init./meas. precision: can "shift" errors

→ systematic errors can be systematically undone (e.g. NMR: "always on" interactions – can be cancelled)

→ not too much correlations in random errors!

Two ess. types of errors:

Dephasing: $|0\rangle + e^{i\phi}|1\rangle$
(Decoherence)

$\Rightarrow$ ϕ randomized

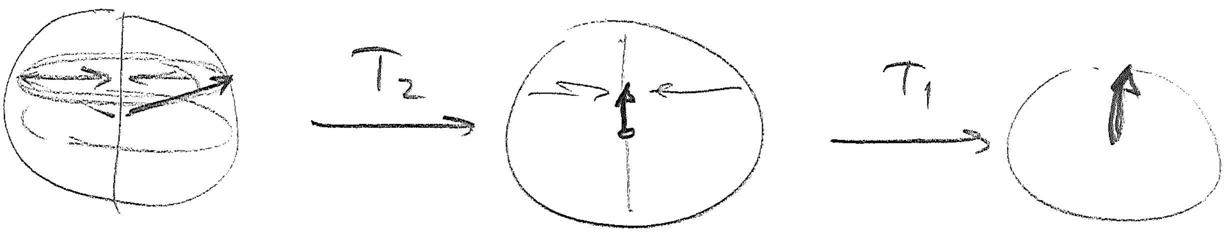
$\Rightarrow$ T_2 time.

Depolarization / Relaxation:

$$\alpha|0\rangle + \beta|1\rangle \longrightarrow |0\rangle$$

$\Rightarrow$ T_1 time

(only if $|0\rangle$ is preferred state, of course!)



By def.: $T_2 < 2T_1$, but typ. $T_2 \ll T_1$.

T_2: relevant for gate operation.

T_1: relevant for (z) measurement.

iv) A universal set of quantum gates

→ Single qubit gates:

→ Rotations in 2 bases

→ often continuous: $U_t = e^{-iHt/\hbar}$

⇒ precise control of t needed!

→ Two qubits gates:

→ one gate (not SWAP) sufficient!

→ best gate architecture-dep.:

$H = \sigma_x \otimes \sigma_x$, $\sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y$, $\vec{\sigma} \cdot \vec{\sigma}$

⇒ all univ. for right time t!

⇒ ideal gate tailored to architecture!

→ Symmetries / constraints:

→ gates can act in effective subspace,

e.g. if H restricted by symmetries

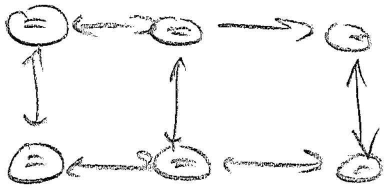
(SU(2), transl. invariance)

→ interactions can be always on & decoupled

by single-qubit ops.

→ Interaction geometry:

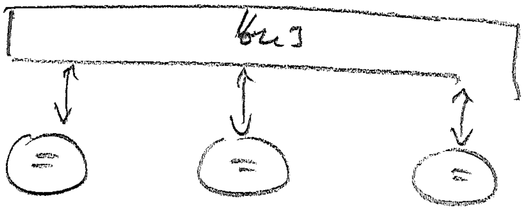
Lattices:



→ dep. on exp. setup (≤ 3D)

→ e.g. solid state

bus:



bus = delocalized quantum DoF
which couples to all qubits

→ need indiv. control over couplings

→ bus more sensitive for bigger systems

e.g.: ion traps: bus = joint vibrations of ions

move qubits:

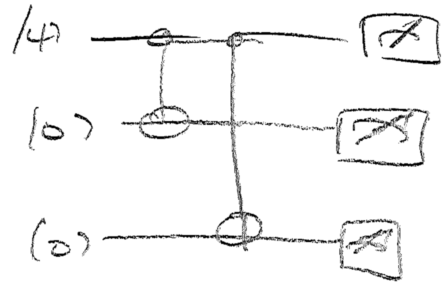
e.g. trapped ions/atoms can be
moved on "conveyor belts" (em-fields
or light fields)

v) A qubit specific measurement capability

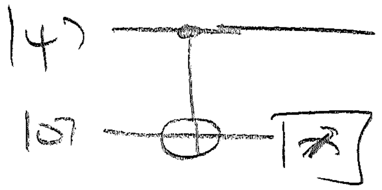
Ideally: Projective meas. in some (Z) basis.

Imperfect measurement:

- ▷ repeat or
- ▷ copy + measure several times



Destructive meas is ok: E.g. via copy:



⇒ non-destructive!

Meas. on 1 qubit is ok: Can swap qubits.

Can be slow ($\sim T_1$) for final meas., but fast ($\ll T_2$) for error correction.

(Note: QECC does not require meas., but computing correction might be much faster on classical computers.)