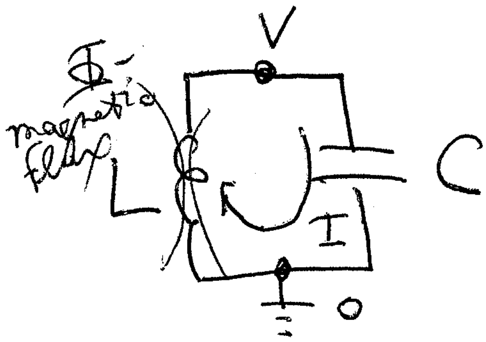
SUPERCONDUCTING QUBITS

- BASIS IN ELECTRIC CIRCUIT THEORY

$$\Phi = -LI$$

$$\dot{\Phi} = V \quad \text{(Faraday)}$$



(*) $Q = CV = C\dot{\Phi}$

$$\dot{Q} = I = C\ddot{\Phi}$$

$$\Rightarrow \boxed{C\ddot{\Phi} = -\frac{1}{L}\Phi}$$ equation of motion

$\rightarrow$ resonates @ $\omega_0 = \frac{1}{\sqrt{LC}}$

Lagrange function $\mathcal{L}(q, \dot{q})$
(conservative mechanics)

equation of motion from Lagrangian:

Euler-Lagrange equation:

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} = \frac{\partial \mathcal{L}}{\partial q}$$

$q \Leftrightarrow \Phi$

canonical coordinate

good guess

$$\mathcal{L} = \underbrace{\frac{1}{2}C(\dot{\Phi})^2}_{\text{electric energy}} - \underbrace{\frac{1}{2L}\Phi^2}_{\text{magnetic energy}}$$

prescription for obtaining Hamiltonian function:

$$p \equiv \frac{\partial \mathcal{L}}{\partial \dot{q}} = \frac{\partial \mathcal{L}}{\partial \dot{\Phi}} = C\dot{\Phi}$$

C like a mass

Legendre transformation

$H = \dot{q}p - \mathcal{L}$, plus replace $\dot{q} = f(p)$

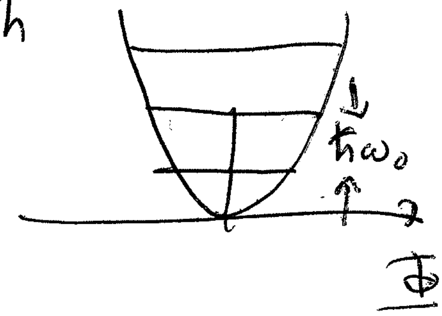
$\dot{\Phi} = \frac{1}{C}p$ NB $p = Q$ from (*)

$$H = \tfrac{1}{2} C \dot{\Phi}^2 + \frac{1}{2L} \Phi^2$$

$$\boxed{H = \frac{Q^2}{2C} + \frac{1}{2L} \Phi^2}$$

quantum postulate: $[\hat{Q}, \hat{\Phi}] = -i\hbar$

$$\hat{H} = \frac{-\hbar^2}{2C} \frac{\partial^2}{\partial \Phi^2} + \frac{1}{2L} \Phi^2 \longrightarrow$$



To make a qubit, make non-harmonic

$\Longrightarrow$ nonlinear inductance

$I = \frac{1}{L} \Phi$ linear

$= f(\Phi)$ nonlinear $\rightarrow$ still non-dissipative

particular nonlinearity in superconducting tunneling device:

Josephson Junction:

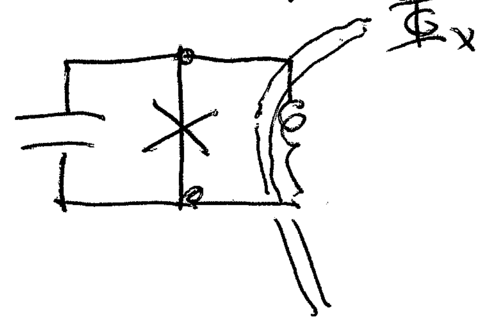
$$I = \frac{\tilde{\Phi}_0}{L_J} \sin(\Phi / \tilde{\Phi}_0) \qquad \tilde{\Phi}_0 = \frac{\hbar}{2e}$$

usually just h.

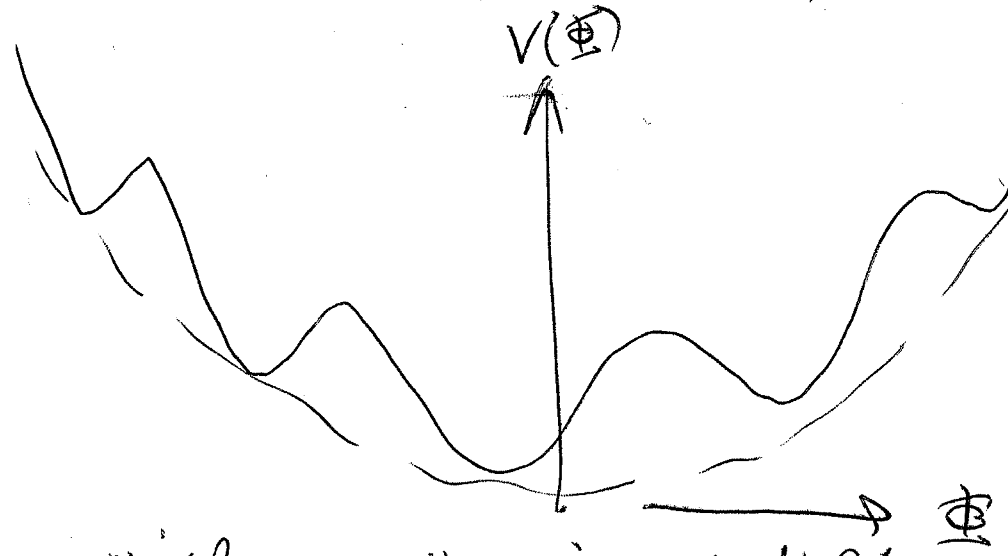
"Josephson inductance"

$\frac{\tilde{\Phi}_0}{L_J} = I_c$ critical current

So, eg. Hamiltonian for



$$\hat{H} = -\frac{\hbar}{2C}\frac{\partial^2}{\partial \Phi^2} + \frac{\tilde{\Phi}_0^2}{L_J}\cos\left(\frac{\Phi}{\tilde{\Phi}_0}\right) + \frac{1}{2L}\Phi^2 + \underbrace{\frac{\Phi\Phi_x}{L}}_{(*)}$$



Highly non-harmonic potential!

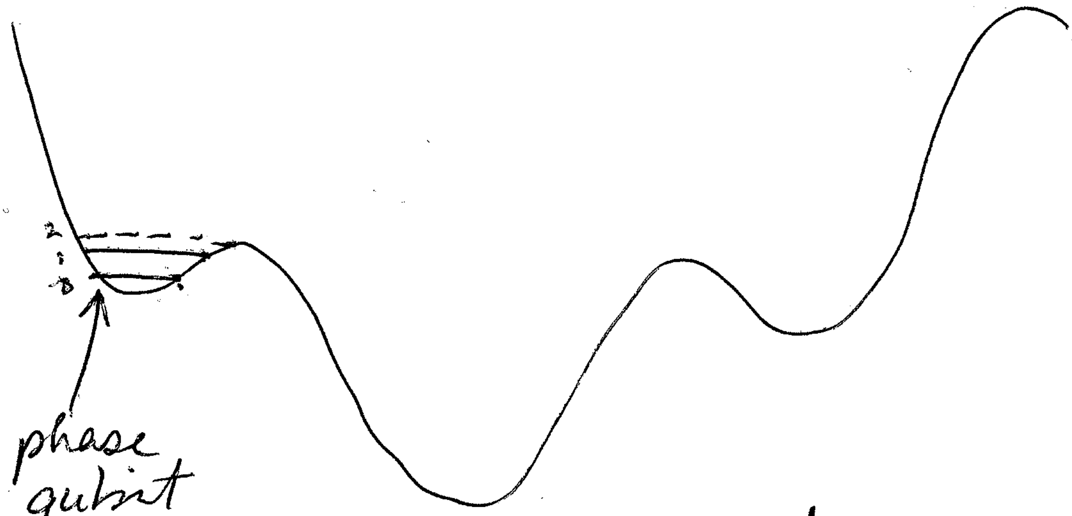
Linear term (*) → characteristic "phase rigidity" – characteristic of superconductivity (not true of "perfect conductor")

MORE GENERAL THEORY FOR DERIVING CIRCUIT HAMILTONIAN:

Network Graph Theory.

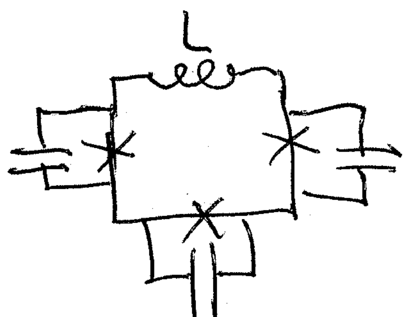
Applications:

1. Phase qubit — Eq. (*) on p. 3.



- use metastable levels — initialize by ramping Φ_x
- read out by pumping to metastable state $|2\rangle$, observe "escape"

2. Flux qubit

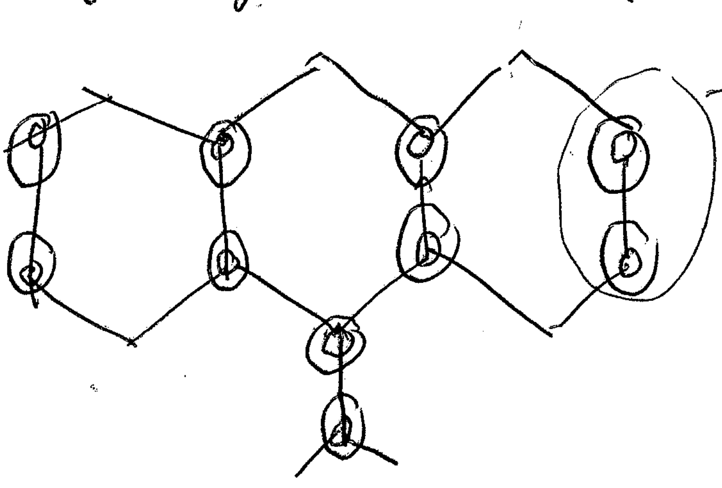


3D Hamiltonian

$$U(\Phi) = \tilde{\Phi}_0^2 \left[\sum_{i=1}^{3} L_{J,i} \cos\varphi_i + \frac{1}{2L}(\varphi_1+\varphi_2+\varphi_3)^2 + \frac{1}{L}(\varphi_1+\varphi_2+\varphi_3)\frac{\Phi_x}{\Phi_0} \right]$$

$\langle 111 \rangle$ direction very tightly confined.

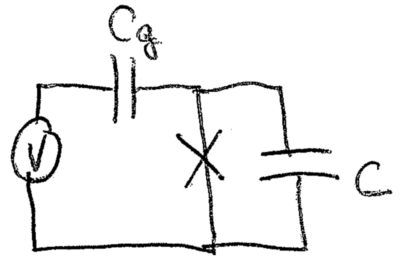
- Potential in plane $\perp$ to $\langle 111 \rangle$:



very controllable double-well potential

TRANSMON QUBIT

& COUPLED SUPERCONDUCTING QUBITS.



see:

www.qipc2011.ethz.ch/uploads/schoolpresentations/qipcAlexandre.pdf

$$H = \frac{1}{2(C+C_g)}\left(\hat{Q} - C_g V\right)^2 - \left(\frac{\Phi_0}{2\pi}\right)^2 \frac{1}{L_J} \cos\hat{\varphi}$$

$$= \underbrace{\frac{(2e)^2}{2(C+C_g)}}_{4E_c}\Big(\hat{n} - \underbrace{\frac{C_g V}{2e}}_{n_{offset}}\Big)^2 - \underbrace{\left(\frac{\Phi_0}{2\pi}\right)^2 \frac{1}{L_J}}_{E_J} \cos\hat{\varphi} \qquad \hat{n} = -i\frac{\partial}{\partial\varphi}$$

$$H = 4E_c(\hat{n} - n_{offset})^2 - E_J \cos\varphi$$

<u>NOTE</u> φ is only defined on interval $0 \le \varphi < 2\pi$.

Periodic boundary conditions if $n_{offset} = 0$ (or integer)

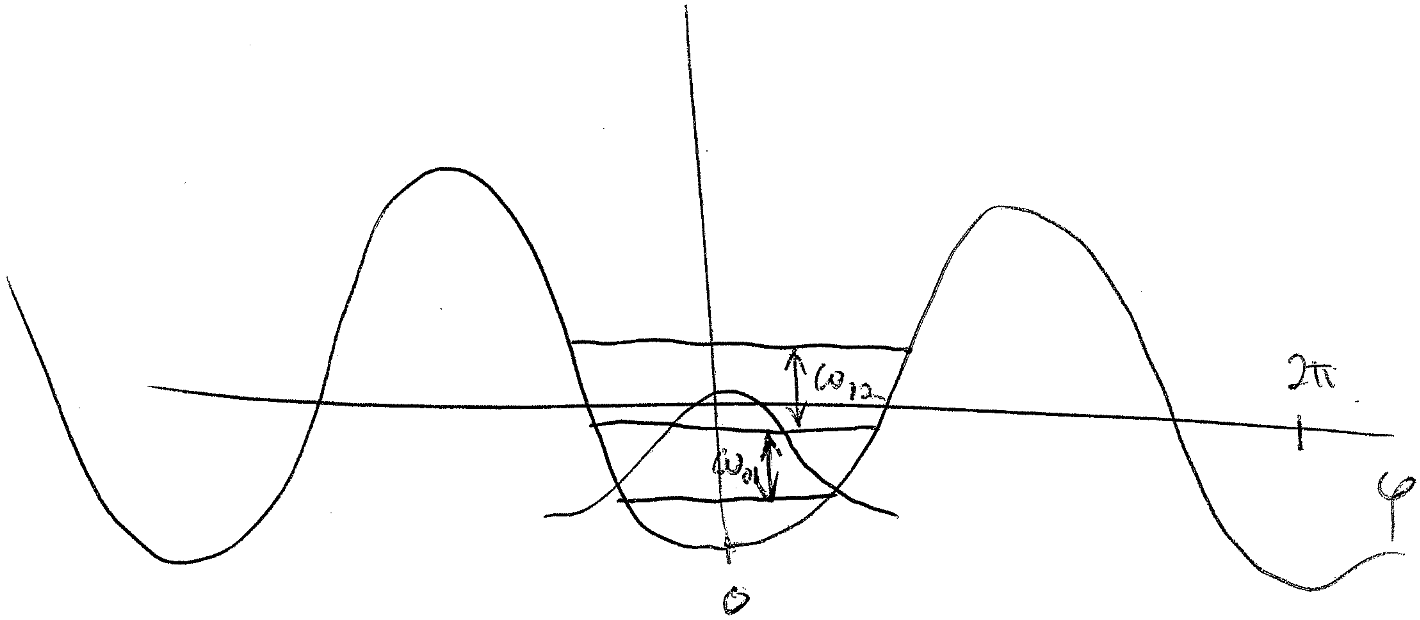
$n_{offset} \neq 0$ acts like Aharonov-Bohm flux

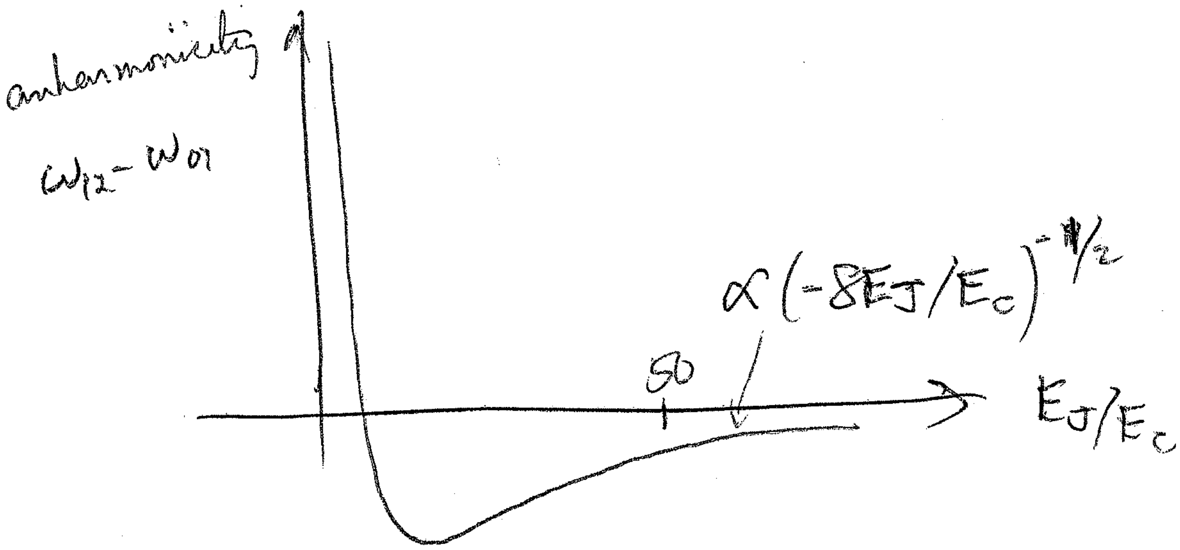
For $E_J = 0$, eigenfunctions are plane waves:

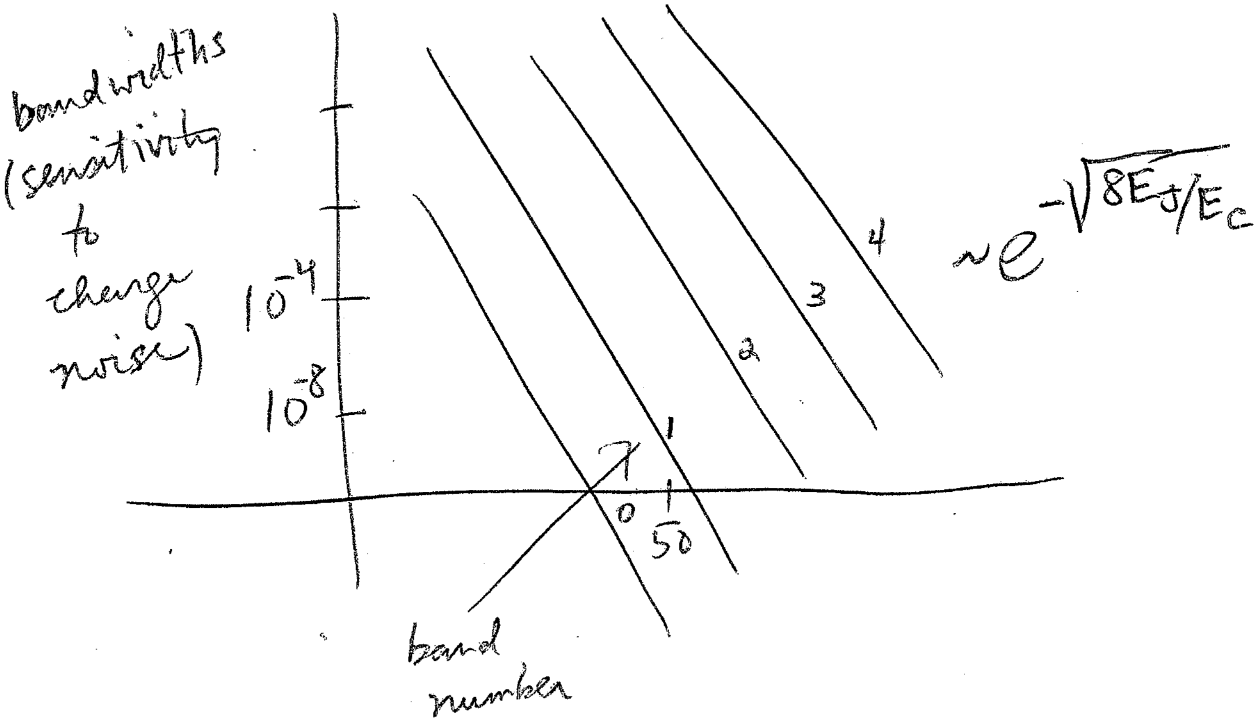
$$H|\psi\rangle = E|\psi\rangle$$

$$|\psi\rangle = e^{im\varphi} \qquad m = 0, \pm1, \pm2, \dots$$

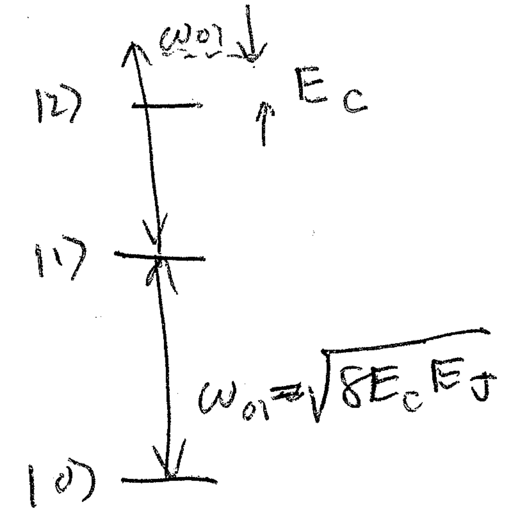
$$E = 4E_c(m - n_{offset})^2$$


ω_{12}
ω_{01}
2π
φ
0


anharmonicity α
$\omega_{12}-\omega_{01}$
$\propto(-8E_J/E_C)^{-1/2}$
50
E_J/E_C


bandwidths (sensitivity to charge noise)
10^{-4}
10^{-8}
$\sim e^{-\sqrt{8E_J/E_C}}$
4
3
2
1
0
50
band number

i.e., charge sensitivity exponentially suppressed, anharmonicity "reasonable".

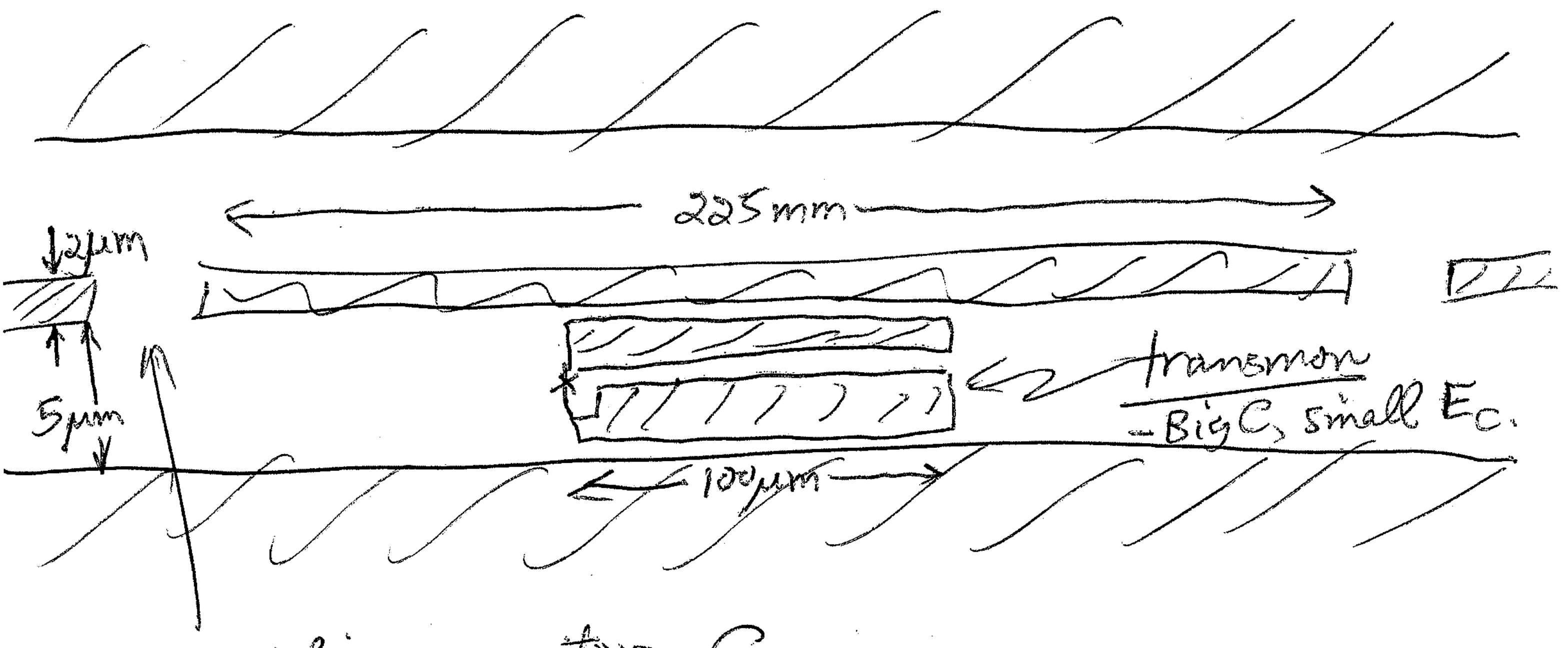


- Very accurate 1-qubit gates,

$$F = 99.993\%$$

- COUPLING TRANSMONS.

coplanar waveguide:

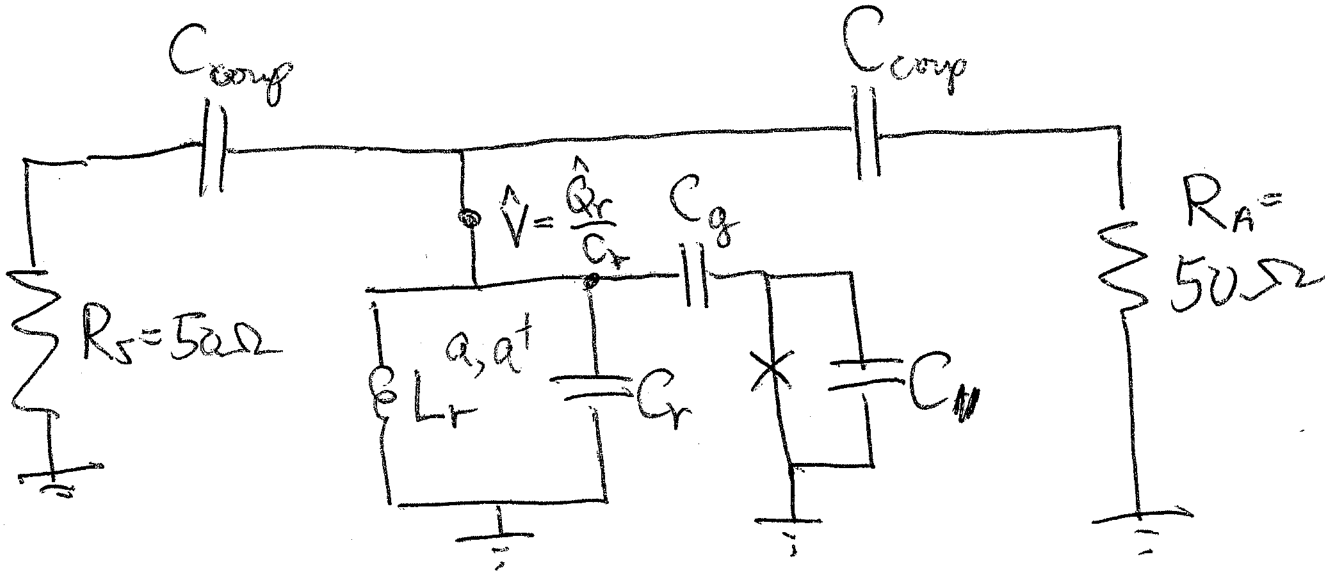


coupling capacitors C_{coup}.

→ slightly transparent mirror for microwaves

CPW is cavity for microwave photons, $\frac{\omega_r}{2\pi} = 2\text{-}20$ GHz.

Circuit model:



R_S, R_A: source & amplifier impedances

L_r, C_r: model for first resonant mode of transmission line resonator

circuit Hamiltonian

$$H = \frac{\hat{Q}_r^2}{2C_r} + \frac{\hat{\Phi}_r^2}{2L_r} + \left[4E_c(\hat{n} - \hat{n}_{offset})^2 - E_J \cos\hat{\varphi}\right]$$

$$\hat{n}_{offset} = \frac{C_g}{2e}\hat{V} = \frac{C_g}{2eC_r}\hat{Q}_r \leftarrow \text{offset-charge operator}$$

coupling term $-8E_c\,\hat{n}\hat{n}_{offset}$

2^{nd} quantize:

rotating-wave approx.

$$H = \omega_r a^\dagger a + \frac{\omega_{01}}{2}Z + g\,(a^\dagger\sigma_- + a\sigma_+)$$

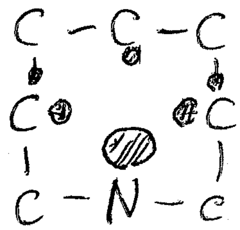
Jaynes-Cummings model.

SPIN QUBITS

- There is a wide variety of spin-qubits.
- We mean <u>electron</u> spin ~~qubits~~.
- We discuss several cases, and consider "which electron". Because electrons are indistinguishable, we really mean "which fermionic mode"

EXAMPLES

1 NV$^-$ center (diamond)



HOMO
highest occupied molecular orbital

vacuum

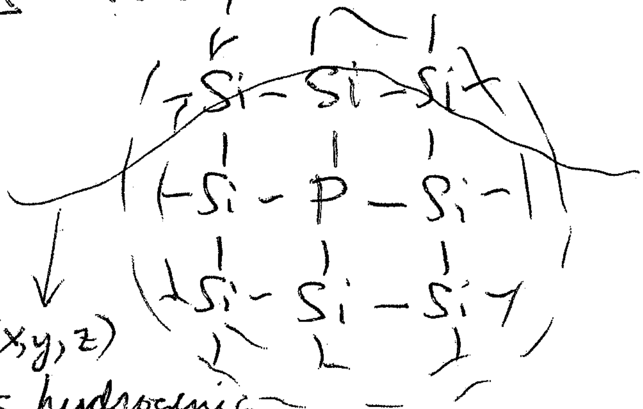
$$\left. \begin{array}{l} c^\dagger_{HOMO(1)\uparrow}\, c^\dagger_{HOMO(2)\uparrow} \\ \text{or} \\ c^\dagger_{HOMO(1)\downarrow}\, c^\dagger_{HOMO(2)\downarrow} \end{array} \right\} \prod_k c^\dagger_k |0\rangle$$

2 of the 3 states of a triplet

"filling the bands"

- orbitals not varied by $\vec{E}$ field

2 "Kane qubit" (B. Kane, 199[illegible])



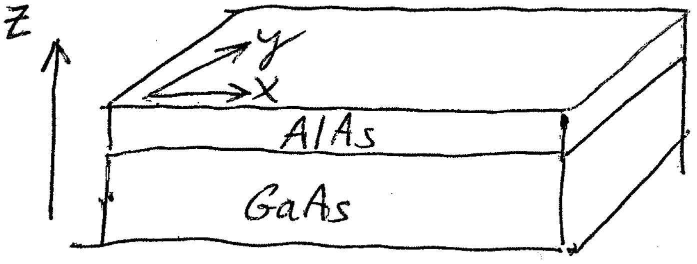
$\varphi(x,y,z)$
"1s hydrogenic orbital"

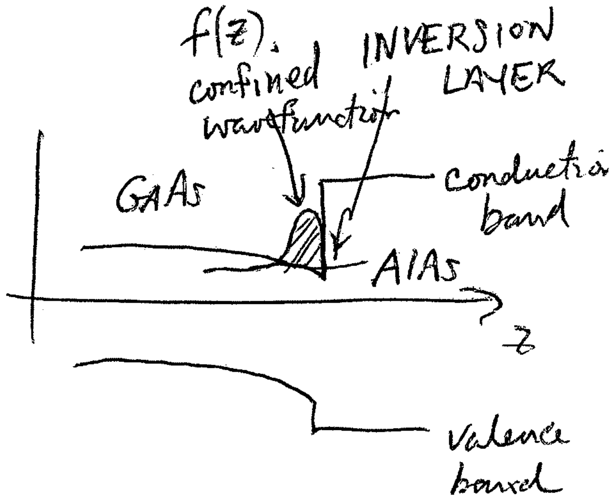
$$|0\rangle = c^\dagger_{S\uparrow} \prod_k c^\dagger_k |0\rangle$$

$$|1\rangle = c^\dagger_{S\downarrow} \prod_k c^\dagger_k |0\rangle$$

- orbital can be distorted by $\vec{E}$ field (large Stark effect)

GaAs QUANTUM-DOT QUBIT



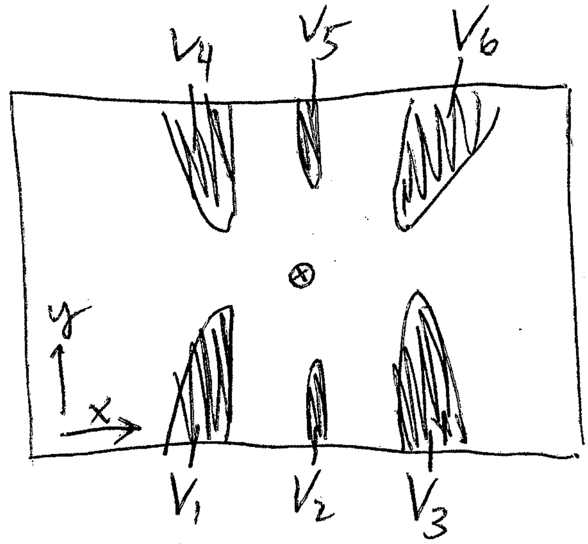


- SIMPLE, CUBIC CRYSTAL STRUCTURE
- defect-free interface
- electrons strongly confined in z-direction, completely free in x and y.
- full wavefunction:

$$\Psi(x,y,z) = f(z)\underbrace{e^{i\vec{k}\cdot(x,y)}\,u(x,y)}_{\text{2D Bloch theorem}}$$

periodic function (crystal period)

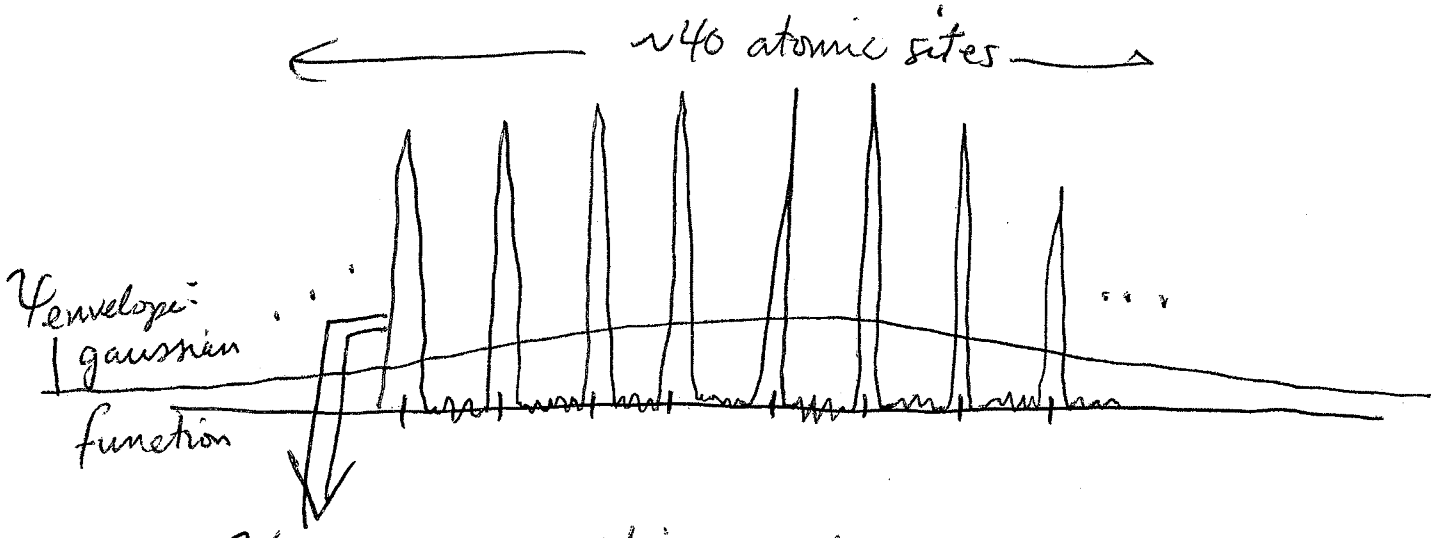
MAKE A QUANTUM DOT BY FURTHER CONFINEMENT:



⊗ minimum of electric potential in interfacial plane now the wavefunction is

$$\Psi(x,y,z) = f(z)\,\Psi_{envelope}(x,y)\,u(x,y)$$

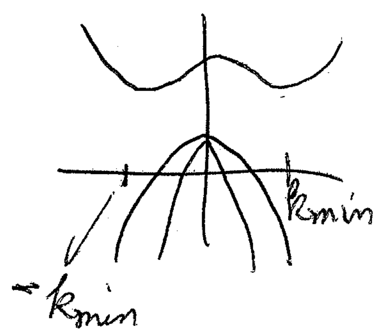
for GaAs, the lowest-energy conduction-band state is at $k=0$ ("direct bandgap"). This leads to a $\Psi_{envelope}$ which is very slowly varying, something like

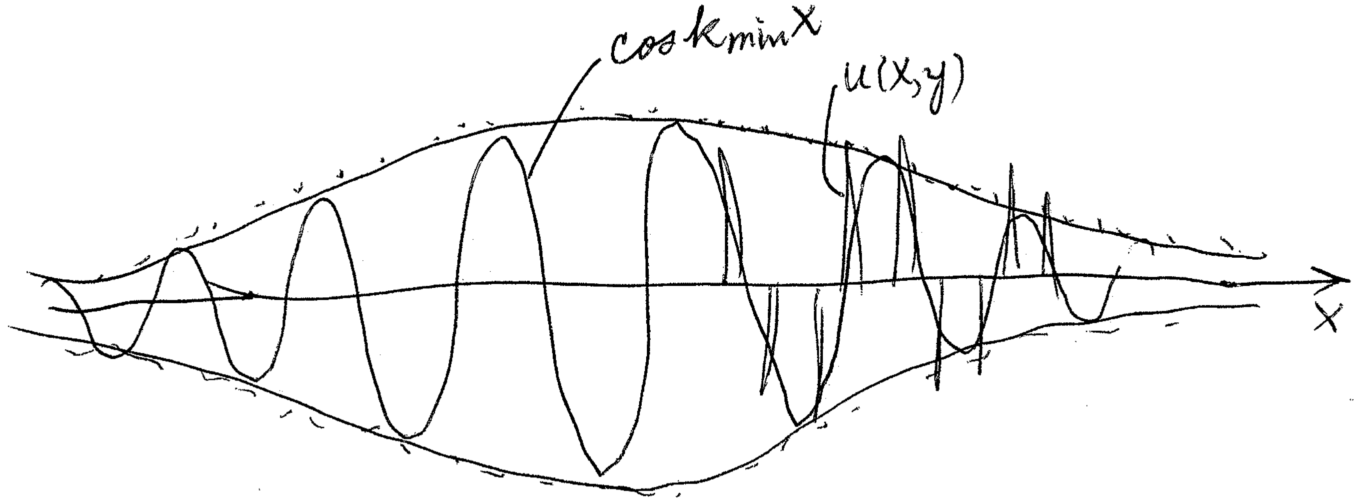


$\Psi_{envelope}$ multiplies the very sharply peaked $u(x,y)$. This function is very large at the site of the nuclei, giving these "concentration factors":

$$\left|\frac{\Psi(0)}{\Psi_{envelope}(0)}\right|^2 = \begin{cases} 2.7\times10^3 & \text{Ga} \\ 4.5\times10^3 & \text{As} \\ 186 & \text{Si} \end{cases}$$

in silicon heterostructures, the wavefunction is less spread out, because the effective mass is heavier. The other complication of Si is that the conduction band minimum is not at $k=0$ ("indirect bandgap"). This causes the envelope function to be more complex:





$g(x) \cos k_{min} x$ and $g(x) \sin k_{min} x$ are two different, almost degenerate wavefunctions. This is "valley degeneracy" (also in y direction), which is not a desirable extra quantum number for a qubit.

NUCLEAR SPINS

^{69}Ga	60%	$I = 3/2$
^{71}Ga	40%	$I = 3/2$
^{75}As	100%	$I = 3/2$
^{28}Si	~90%	$I = 0$
^{30}Si	~6%	$I = 0$
^{29}Si	~3%	$I = 1/2$
^{12}C	99%	$I = 0$
^{13}C	1%	$I = 0$
^{229}Th	?	$I = 5/2$ $I = 3/2$ ← extra nuclear level 7.8±0.5eV above ground state

^{14}N	99.6%	$I=1$
^{15}N	0.4%	$1/2$

amplification factor is important because of <u>Fermi contact hyperfine interaction</u>:

$$H_{SB} = \sum_{i=1}^{10000} A|\psi(\vec{r}_i)|^2 \vec{S} \cdot \vec{I}_i$$

electron spin   nuclear spin

if all nuclear spins are polarized, $I_i = -\frac{3}{2}$, the quantity

$$\sum_{i=1}^{10000} A|\psi(r_i)|^2 \left(-\frac{3}{2}\right)$$

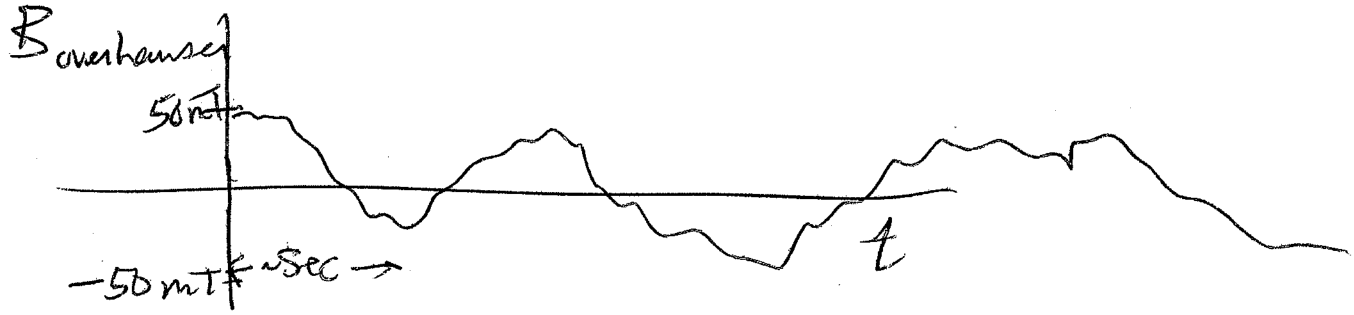
is like a z magnetic field of 5T.

(Overhauser field)

N.B. of the three terms in the hyperfine hamiltonian, the "longitudinal term" $Z I_z$ is 1st order in perturbation theory, the "transverse terms" $X I_x + Y I_y$ are 2nd order. The transverse terms also disappear in a rotating wave approximation.

Actual situation with nuclear spins:

Z-magnetization is a fluctuation around a zero mean, with amplitude $O(\sqrt{N})$: $\frac{5T}{\sqrt{N}} \sim 50mT$

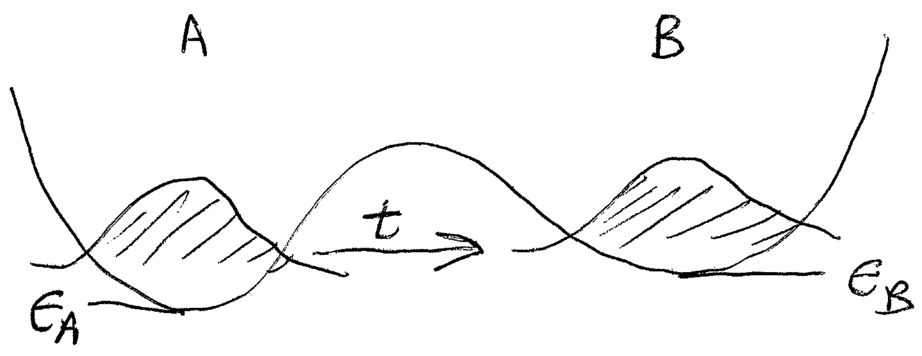


$T^* \sim 15mT$, can be effectively refocussed.

NEXT: 2 quantum dots.

SPIN QUBITS – CONTINUED.

– TWO COUPLED SPINS:



2^{nd} QUANTIZED HAMILTONIAN:

$$H = \epsilon_A (a_\uparrow^\dagger a_\uparrow + a_\downarrow^\dagger a_\downarrow) + \epsilon_B (b_\uparrow^\dagger b_\uparrow + b_\downarrow^\dagger b_\downarrow) +$$
$$t\,(a_\uparrow^\dagger b_\uparrow + b_\uparrow^\dagger a_\uparrow + a_\downarrow^\dagger b_\downarrow + b_\downarrow^\dagger a_\downarrow) +$$
$$U_A\, a_\uparrow^\dagger a_\uparrow a_\downarrow^\dagger a_\downarrow + U_B\, b_\uparrow^\dagger b_\uparrow b_\downarrow^\dagger b_\downarrow$$

Work in Fock space of 2 electrons in these 4 modes

$$|\underset{a_\downarrow}{\uparrow}, \underset{a_\uparrow}{\uparrow}, \underset{b_\downarrow}{\uparrow}, \underset{b_\uparrow}{\uparrow}\rangle \qquad \binom{4}{2} = 6 \text{ Dimensional space}$$

$$H = \begin{array}{c|cccc|cc}
 & 0101 & 0110 & 1001 & 1010 & 1100 & 0011 \\
\hline
0101 & \epsilon_A+\epsilon_B & & 0 & & 0 & 0 \\
0110 & & \epsilon_A+\epsilon_B & & & -t & -t \\
1001 & & 0 & \epsilon_A+\epsilon_B & & t & t \\
1010 & & & & \epsilon_A+\epsilon_B & 0 & 0 \\
\hline
1100 & 0 & -t & t & 0 & U_A+2\epsilon_A & 0 \\
0011 & 0 & -t & t & 0 & 0 & U_B+2\epsilon_B
\end{array}$$

a non-trivial 2-qubit gate requires that we include the effects of the non-qubit states $|1100\rangle$ and $|0011\rangle$

- Sign of t:

These signs arise from the antisymmetry of the Fermionic wavefunction. Such signs are ~~···~~ inevitable, since if we call

$$|0110\rangle \equiv a_\uparrow^\dagger b_\downarrow^\dagger |0000\rangle$$

then $b_\downarrow^\dagger a_\uparrow^\dagger |0000\rangle = -|0110\rangle$, because of the anticommutation relations of the creation and destruction operators. ~~These~~ Everything works properly if you use this standard, general convention for the sign:

$$c_x^\dagger |b_1, b_2, \ldots b_{x-1}, \boxed{0}, \ldots\rangle = (-1)^{(b_1+b_2+b_3+\ldots b_{x-1}) \bmod 2} |b_1, b_2 \ldots b_{x-1}, \boxed{1}, \ldots\rangle$$

mode x

$$c_x |b_1 b_2 \ldots \boxed{1}, \ldots\rangle = (-1)^{(b_1+b_2+\ldots b_{x-1}) \bmod 2} |b_1, b_2, \ldots b_{x-1}, \boxed{0}, \ldots\rangle$$

With this the evaluation of the matrix elements gives 2 types of cases

no electron "in the way"

$$\langle 1,0,0,0|H|0,0,1,0\rangle = +t$$

electron "in the way"

$$\langle 1,1,0,0|H|0,1,1,0\rangle = -t.$$

Interpreting our 6x6 Hamiltonian.

– Basis change

$$|S\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

$$|T_0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$$

<u>notation</u>

$\uparrow\downarrow \Leftarrow 0110$

$\downarrow\uparrow \Leftarrow 1001$

$$H: \quad \begin{array}{c|cccc|cc}
\uparrow\uparrow & & & & & 0 & 0 \\
S & & & 0 & & -\sqrt{2}t & -\sqrt{2}t \\
T_0 & & & & & 0 & 0 \\
\downarrow\downarrow & & & & & 0 & 0 \\
\hline
1100 & 0 & -\sqrt{2}t & 0 & 0 & U_A - \epsilon_B + \epsilon_A & 0 \\
0011 & 0 & -\sqrt{2}t & 0 & 0 & 0 & U_B + \epsilon_B - \epsilon_A
\end{array}$$

NB: I have shifted the zero of energy by $\epsilon_A + \epsilon_B$.

2^{nd} order perturbation theory shifts the eigenvalue of $|S\rangle$ down by

$$\langle S|H|S\rangle = \frac{-2t^2}{U_A + \epsilon_A - \epsilon_B} - \frac{2t^2}{U_B + \epsilon_B - \epsilon_A} \equiv J$$

truncated Hamiltonian:

$$H = \begin{array}{c} \uparrow\uparrow \\ \text{"}S\text{"} \\ T_0 \\ \downarrow\downarrow \end{array} \begin{pmatrix} 0 & & & \\ & J & & 0 \\ & & 0 & \\ & 0 & & 0 \end{pmatrix}$$

"$|S\rangle$" contains some perturbative admixture of $|1100\rangle$ and $|0011\rangle$

– going to original basis:

$$H = \begin{array}{c} \uparrow\uparrow \\ \text{"}\uparrow\downarrow\text{"} \\ \text{"}\downarrow\uparrow\text{"} \\ \downarrow\downarrow \end{array} \begin{pmatrix} 0 & & & \\ & \frac{1}{2}J & -\frac{1}{2}J & \\ & -\frac{1}{2}J & \frac{1}{2}J & \\ & & & 0 \end{pmatrix}$$

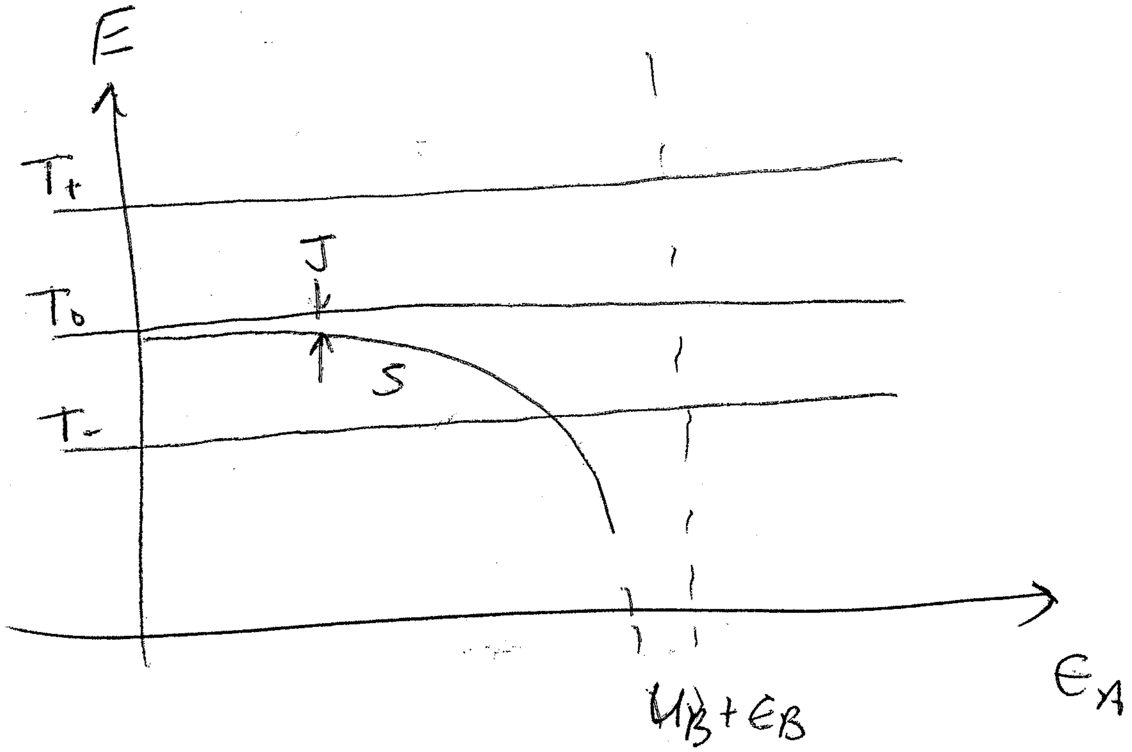
NB $\quad XX + YY + ZZ - II = \begin{pmatrix} 0 & & & \\ & -2 & 2 & \\ & 2 & -2 & \\ & & & 0 \end{pmatrix}$

Thus, up to another shift,

$H \propto J \vec{S}_A \cdot \vec{S}_B$ Heisenberg interaction

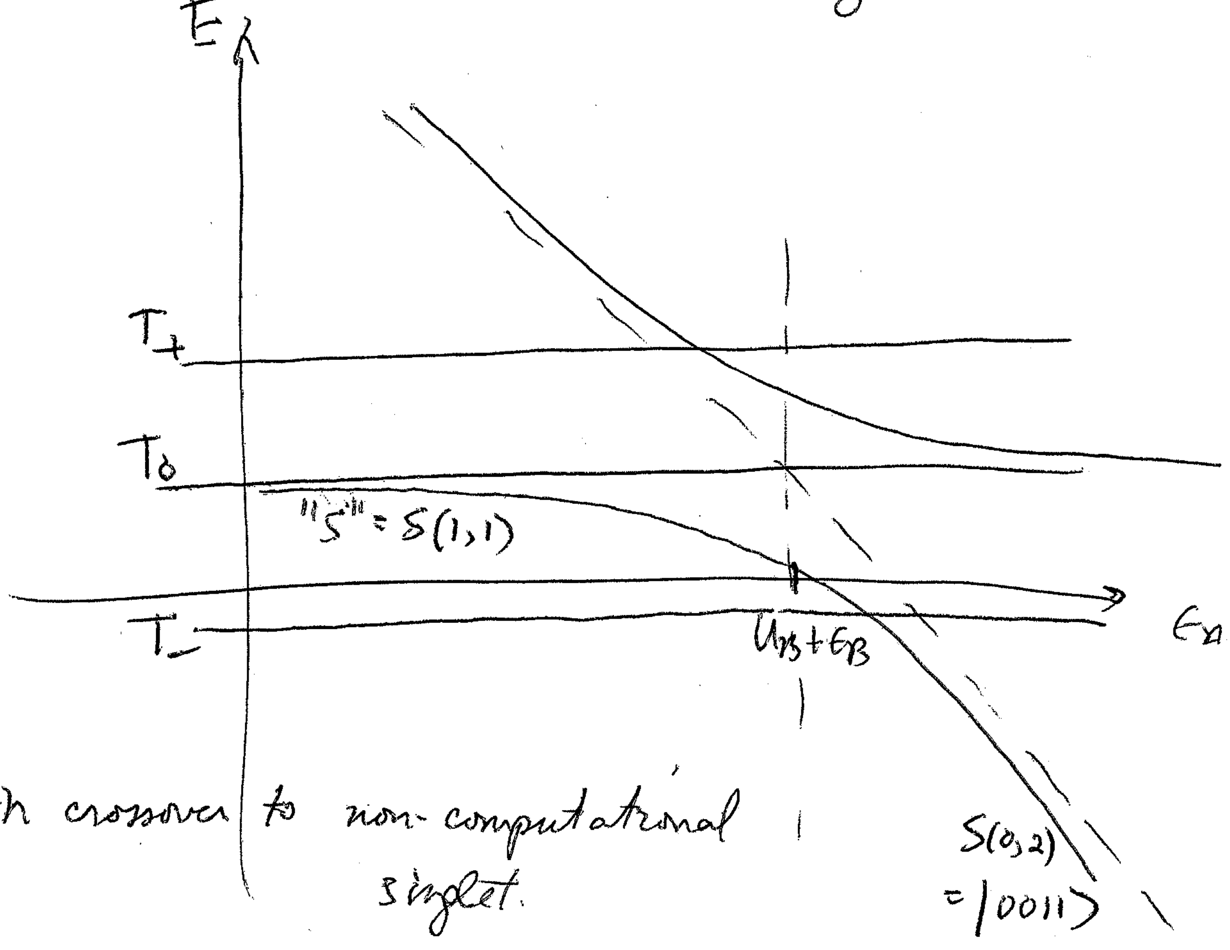
– consider J as a function of ϵ_A:

ADDING ZEEMAN FIELD TO BREAK $|T_{0,\pm}\rangle$ DEGENERACY:



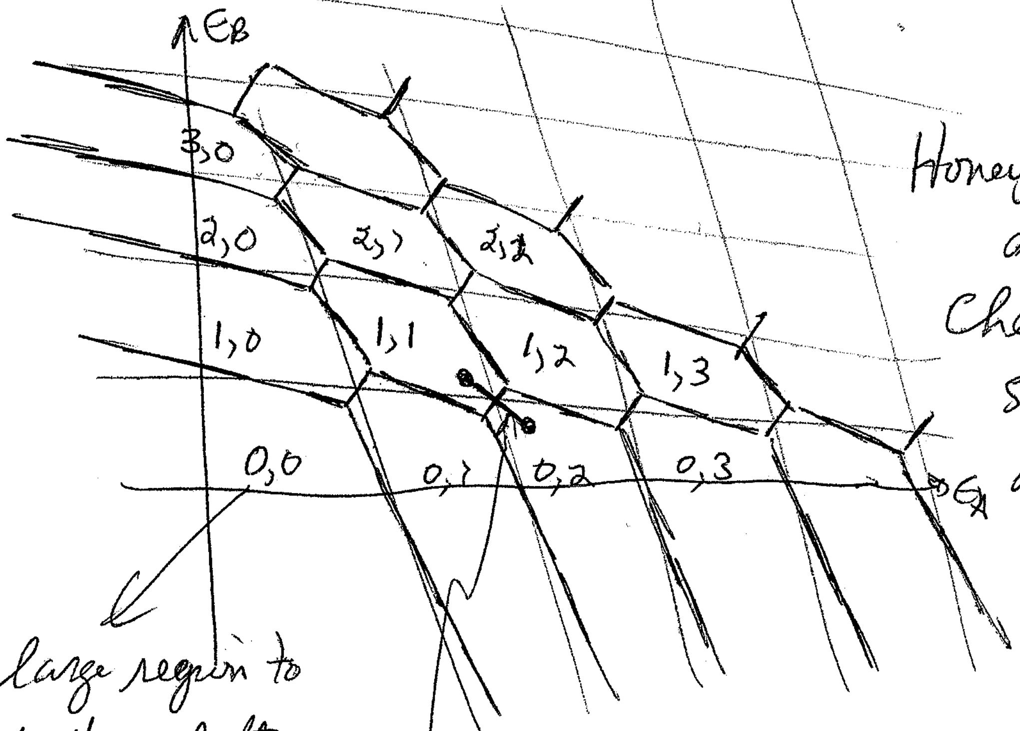
divergence at this point – artifact of perturbation theory.

– DIAGONALIZE 6X6 MATRIX:



Smooth crossover to non-computational singlet.

MORE GLOBALLY:



Honeycomb diagram; Charge-stability diagram

large region to bottom left.

QUANTUM EXPERIMENTS

CHARGE SENSING — POINT CONTACT