Lecture "Analytical and Numerical Methods for Quantum Many-Body Systems from a Quantum Information Perspective" — Exercise Sheet #5

1. Determine the scaling of the approximate PEPS contraction method as a function of the bond dimension D and the dimension αD^2 used in the truncation. Consider the cost for *i)* computing expectation values and *ii)* solving the eigenvalue problem which occurs when minimizing the energy as a function of one $A^{[x,y]}$.

2. Determine the cost of computing *i)* expectation values and *ii)* two-point correlation functions in a (1D) MERA as a function of the bond dimension χ, the system size N, and (for correlations) the distance of the observables.

3. Consider a square lattice of qubits, and, for each vertex i, define the operator

$$S_i := X_i \otimes \bigotimes_{j \in \mathrm{nghb(i)}} Z_j \ ,$$

where X and Z are Paulis, X_i acts on site i, and the four Z_j act on the four sites neighboring i. The *cluster state* is the joint $+1$ eigenstate of all S_i.

Prove that the cluster state can be written as a PEPS with maps $\mathcal{P} = |0\rangle\langle 0,0,0,0| + |1\rangle\langle 1,1,1,1|$, and bonds $|H\rangle = (\mathbb{1} \otimes H)(|00\rangle + |11\rangle) = (H \otimes \mathbb{1})(|00\rangle + |11\rangle)$, with $H = \left(\begin{smallmatrix} 1 & 1 \\ 1 & -1 \end{smallmatrix}\right)/\sqrt{2}$ the Hadamard transformation. (Note that $H^2 = \mathbb{1}$ and $HZ = XH$.)

The cluster state can alternatively also be characterized as follows: Initialize all qubits to $|+\rangle$, and then apply a controlled-phase gate

$$\mathrm{CPHASE} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}$$

to all adjacent vertices (they commute, so the order does not matter). Try to also understand the PEPS description of the cluster state starting from this characterization.

The cluster state can be defined on arbitrary graphs in exactly the same way. Convince yourself that the PEPS representation works for any graph.

4. The *resonating valence bond state* on a square lattice of qubits is the equal weight superposition of all possible ways to fully cover the lattice with singlets. (This is, the singlets are formed between pairs of adjacent spins, and there are no unpaired spins.) Verify that the RVB state can be written as a PEPS with bond $|\omega\rangle = |01\rangle - |10\rangle + |22\rangle$, and

$$\mathcal{P} = |0\rangle\big[|0222\rangle + |2022\rangle + |2202\rangle + |2220\rangle\big] + |1\rangle\big[|1222\rangle + |2122\rangle + |2212\rangle + |2221\rangle\big] \ .$$