The multiscale entanglement renormalization ansatz (MERA)

MPS: very well suited for gapped 1D systems

→ area law

→ exp. decay of correlations (length scale!)

What if system is gapless (i.e. at phase transition)?

→ diverging correlation length

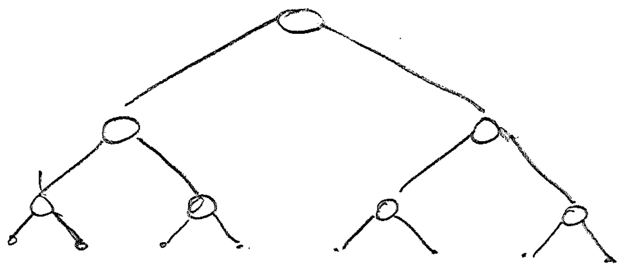
→ system is scale invariant ↔ no length scale

→ correlations algebraic (corr $\sim \ell^{-\alpha}$)

→ Logarithmic scaling of block entropy: $S(\rho_L) \sim \log L$.

Ansatz for scale-invariant systems?

First idea: tree tensor network:

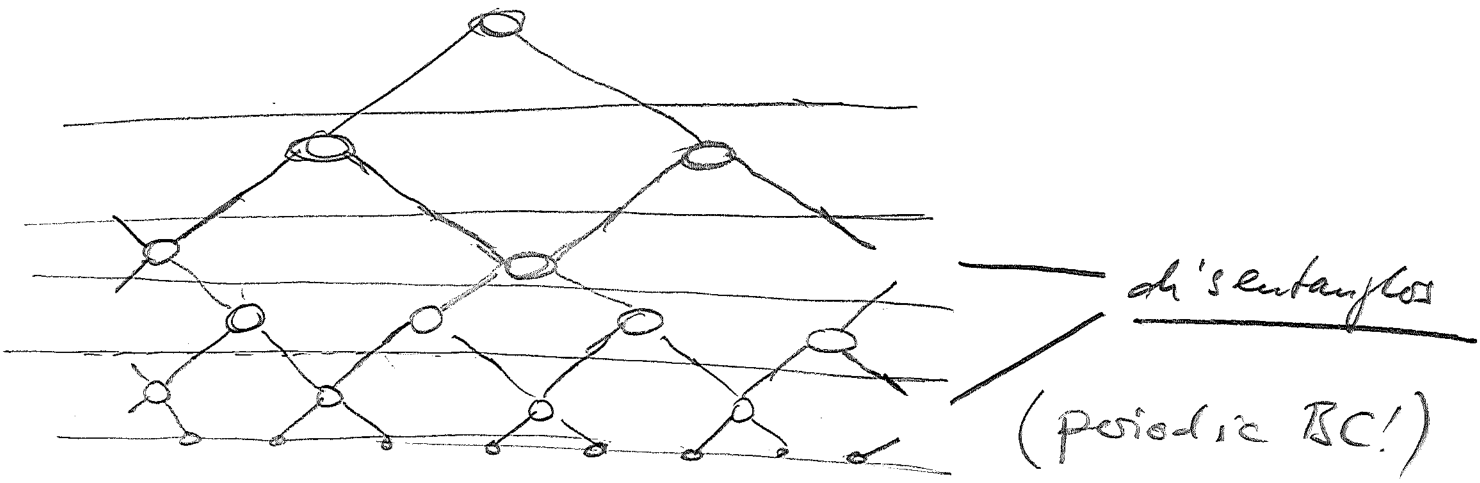


⊕ Has scale invariance.

⊖ Entanglement: still area law scaling

⊖ Entanglement deter. closeby regions dep. on position hard to establish.

→ Include "disentanglers" which disentangle regions:



We will choose the disentanglers to be unitary and the other tensors to be isometries.

"Multiscale entanglement renormalization ansatz" (MERA)

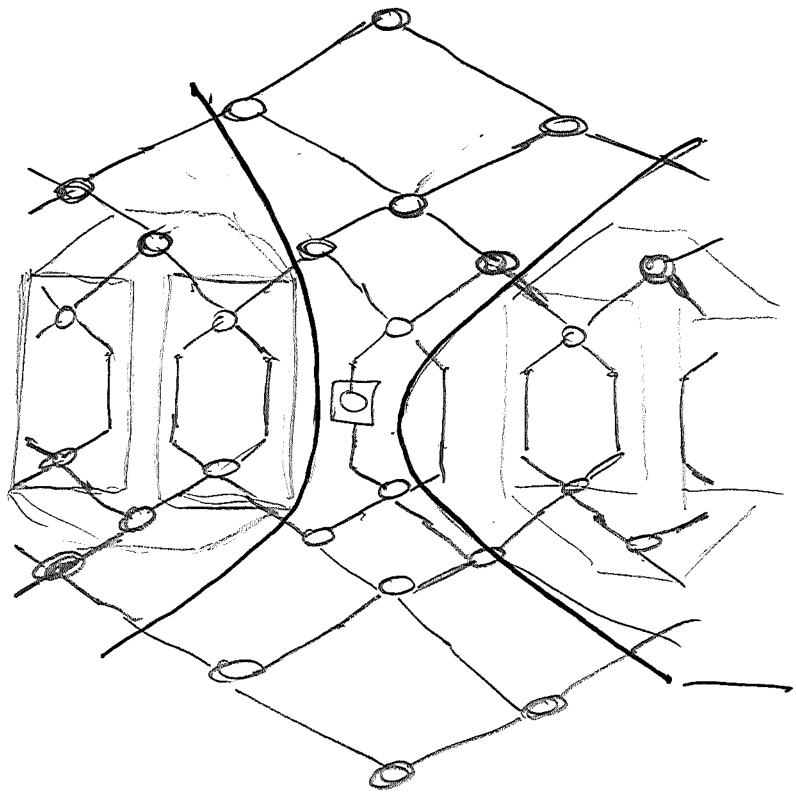
(G. Vidal, quant-ph/0610099)

We use some maximal dimension χ for the tensor indices.

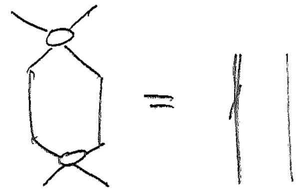
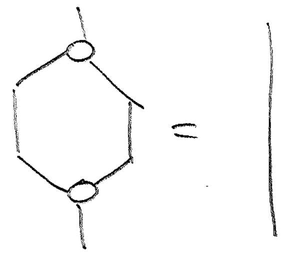
→ Efficient parametrization (in χ and N) of a class of 1D states (w/ built-in scale invariance) — scales like $O(\chi^4 \cdot N)$

By choosing identical tensors in all layers, we obtain a scale-invariant state!

Can we compute exp. values efficiently?



"causal cone" of O

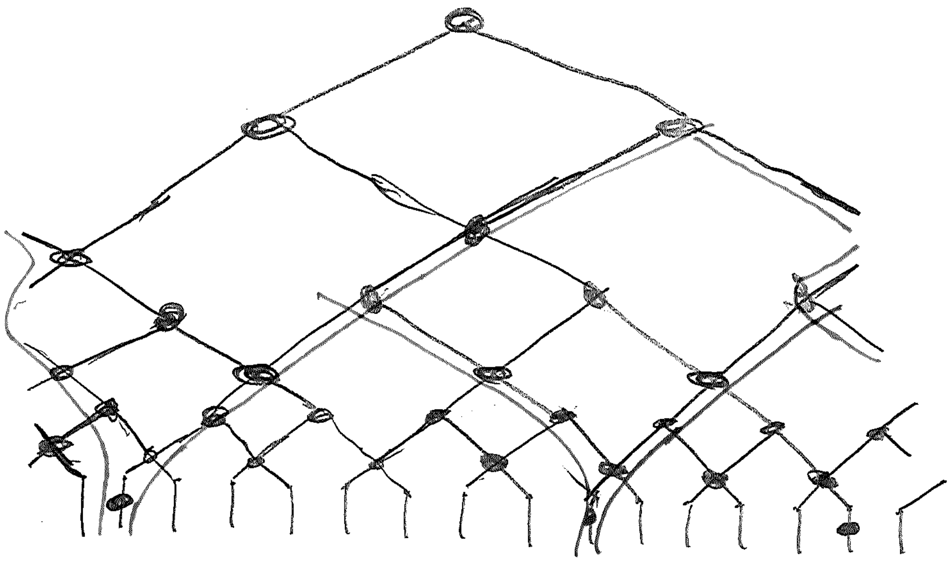
use  and 

to cancel most of the network!

⇒ "causal cone" of operator O has always fixed width (in terms of tensors! – in "real space", the width grows exponentially!), and depth $\log N$.

⇒ Can be contracted efficiently! (Cost: $\log N \cdot \mathrm{poly}(\chi)$)

Correlation functions:



⋮ etc.

two causal cones of fixed width, which eventually fuse to one causal cone

$\Rightarrow$ correl. functions can be computed efficiently!

What scaling should we expect for correl. functions?

- the two causal cones intersect after $\log \ell$ layers. (ℓ: distance)
- If tensors are the same (scale inv.), each layer reduces correlations by some factor λ.

$$\Rightarrow \text{Corr} \sim \lambda^{2\log \ell} = \ell^{2\log \lambda}$$

$\Rightarrow$ algebraic decay of correlations expected!

What entanglement scaling do we expect?

Entanglement $\equiv$ # bonds cut = # layers (depth)

$\Rightarrow$ Ent. should scale like log L!

Simulation w/ MERA:

Variationally optimize energy as function of tensors.

Either: all tensors identical (per layer or all),

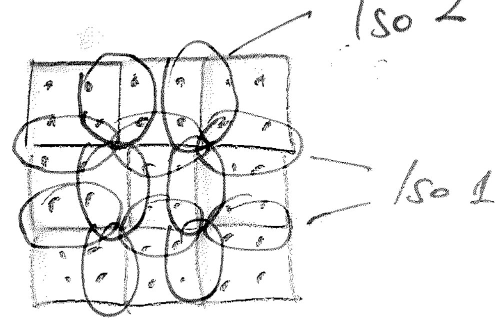
or different tensors everywhere.

Note: Although energy is bilinear in each tensor, this cannot be mapped to eigenvalue problem due to unitarity constraint.

Higher dimension:

Can be generalized to 2D etc.

Idea always: Partition system into blocks, disentangle bdrs. blocks, and reduce # blocks w/ isometries:



Note: Contraction still exact – no approx. needed (cf. PEPS)

Unitary tensor networks:

Basic idea for efficient contraction in MERA:

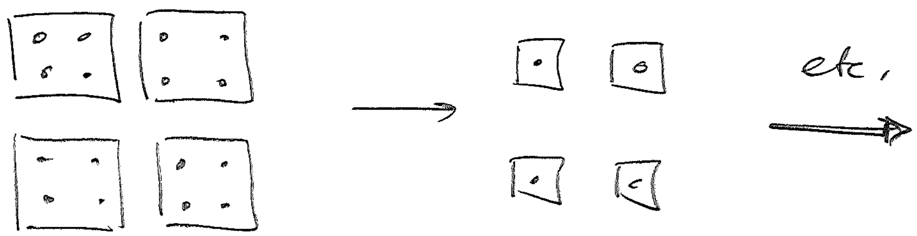
Unitarity of tensors

This can be used to construct a large class of contractible tensor networks where the tensor are unitary!

Relation to renormalization (RG):

Real space renormalization (Kadanoff, Wilson, ...):

Block sites in a system and discard "irrelevant local" degrees of freedom.



E.g.: Ising model: Majority vote!

(This also gives a trafo of Hamiltonian & temperature for the "effective Ham." on renormalized deg. of freedom)

This procedure isolates the long-range features of the system & removes short-range fluctuations.

E.g. Ising model, low temperature: after a few RG steps, all effective spins point in the same direction → RG decreases temperature.

For high temp., the pattern will become more random by RG → T increases.

Critical states/systems: Invariant under RG, since no length scale.

Problem w RG for quantum states:

* To discard irrelevant DoF, we want to isolate them, i.e., they should factor.

* Irrelevant local DoF will however also be entangled across blocks

⟹ need for disentanglers!